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# PERFORMANCE SUMMARY OF THE SUPERSONIC DIFFUSER AND ITS APPLICATION TO ALTITUDE TESTING OF CAPTIVE ROCKET ENGINES

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**AIR FORCE FLIGHT TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE**

Addendum 1 to
AFFTC-TR-60-1
April 1961

PERFORMANCE SUMMARY OF THE SUPERSONIC DIFFUSER
AND I.S APPLICATION TO ALTITUDE TESTING OF
CAPTIVE ROCKET ENGINES

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# ABSTRACT

This report presents the final results of a development program
which was conducted by the Directorate of Rocket Propulsion, Edwards
Air Force Base, California to determine the altitude simulation
produced by various second and third stage rocket engines enclosed
in an altitude chamber and fired into a constant cross-sectional area
diffuser.

The diffuser and supersonic nozzle configurations were tested
utilizing nitrogen gas as the primary fluid; and also with 1000 pound
thrust solid propellant rocket engines. The experimental results are
in accord with the one dimensional equations describing part of the
diffuser performance curve and establish a criteria for comparison
with the results obtained from the 1000 pound thrust solid propellant
rockets. The resulting data presents a method of diffuser application
for captive rocket engine testing at altitudes in excess of 100,000 feet.
The minimum altitude chamber pressure produced with a solid
propellant 15 degree conical Mach 4 configuration was .008 atmospheres
while the minimum altitude chamber pressure produced in the nitrogen
gas tests was .006 atmospheres with a slowly diverging Mach 4
configuration.

Phase II of diffuser development testing was conducted by contractor
personnel at the "Rocket Site" using a two dimensional variable area
contoured diffuser. The results of an optimum series of Mach 3, 4 and 5
test configurations have been included in this report. More detailed
results of these Phase II tests will be available through ASTIA in a
report by the Sandberg-Serrell Corporation.

This report has been reviewed and approved

HAROLD W. NORTON
Colonel, USAF
Director, Rocket Propulsion and Missiles

# table of contents
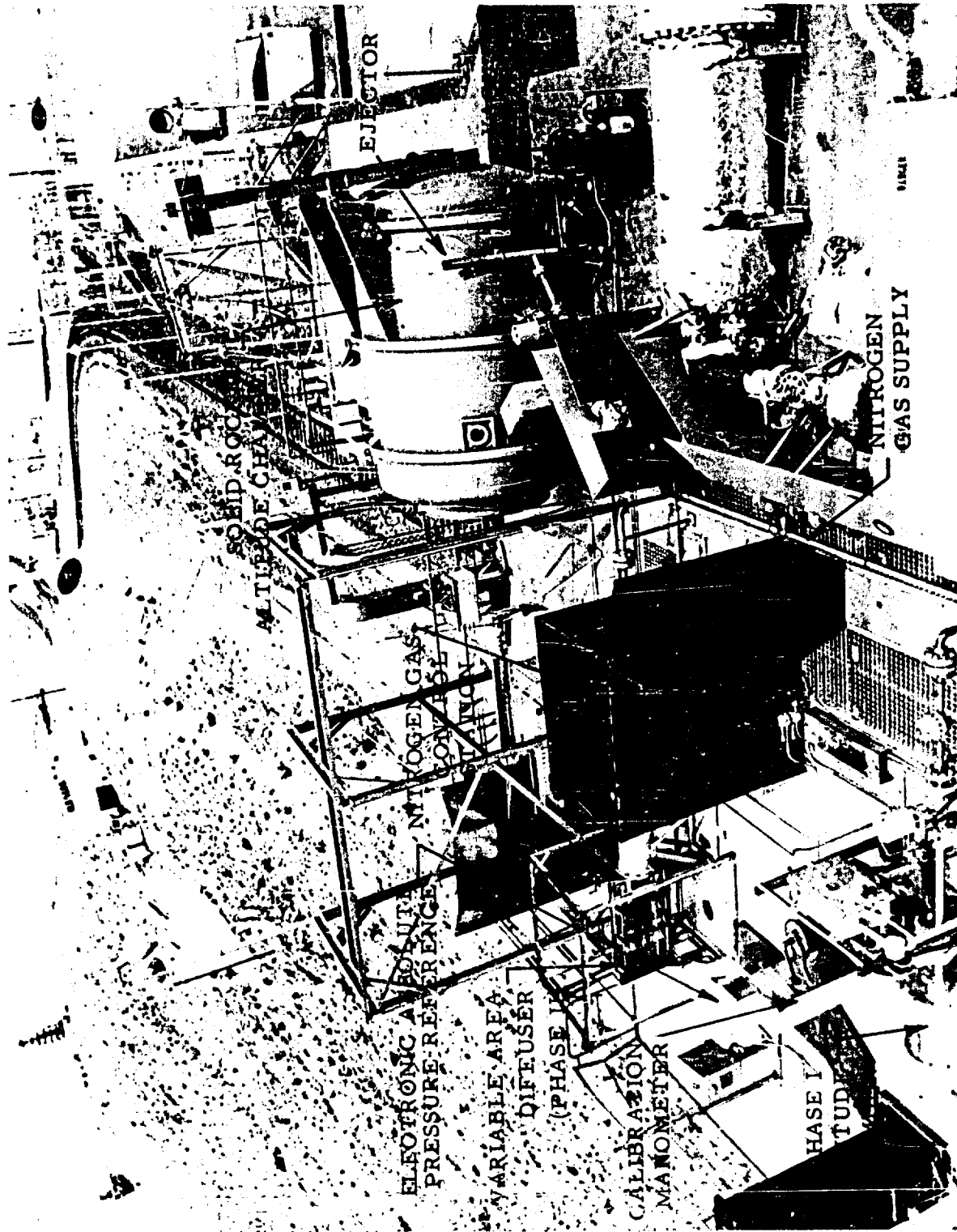
EJECTOR

NITROGEN
GAS SUPPLY

SOLID ROCK
MOTOR ELE CHAMBER

NITROGEN GAS
CONTROL
STATION

ELECTRONIC ABSOLUTE
PRESSURE REFERENCE

VARIABLE AREA
DIFFUSER

(PHASE

CALIBRATION
MANOMETER

HASE I
TUD

# INTRODUCTION

The results of recent supersonic diffuser tests conducted by the Directorate of Rocket Propulsion have qualitatively indicated the influence of a number of geometrical parameters on diffuser performance. This report has attempted to answer some of the basic questions concerning captive testing of rocket engines at altitude and has been limited to the investigation of the principle factors influencing diffuser performance. New questions are evident if requirements for testing at still higher altitudes are brought forth. In anticipation of these requirements, studies are currently being conducted to determine the feasibility of systems capable of maintaining altitudes in excess of 100,000 feet during rocket engine cut-off and restart.
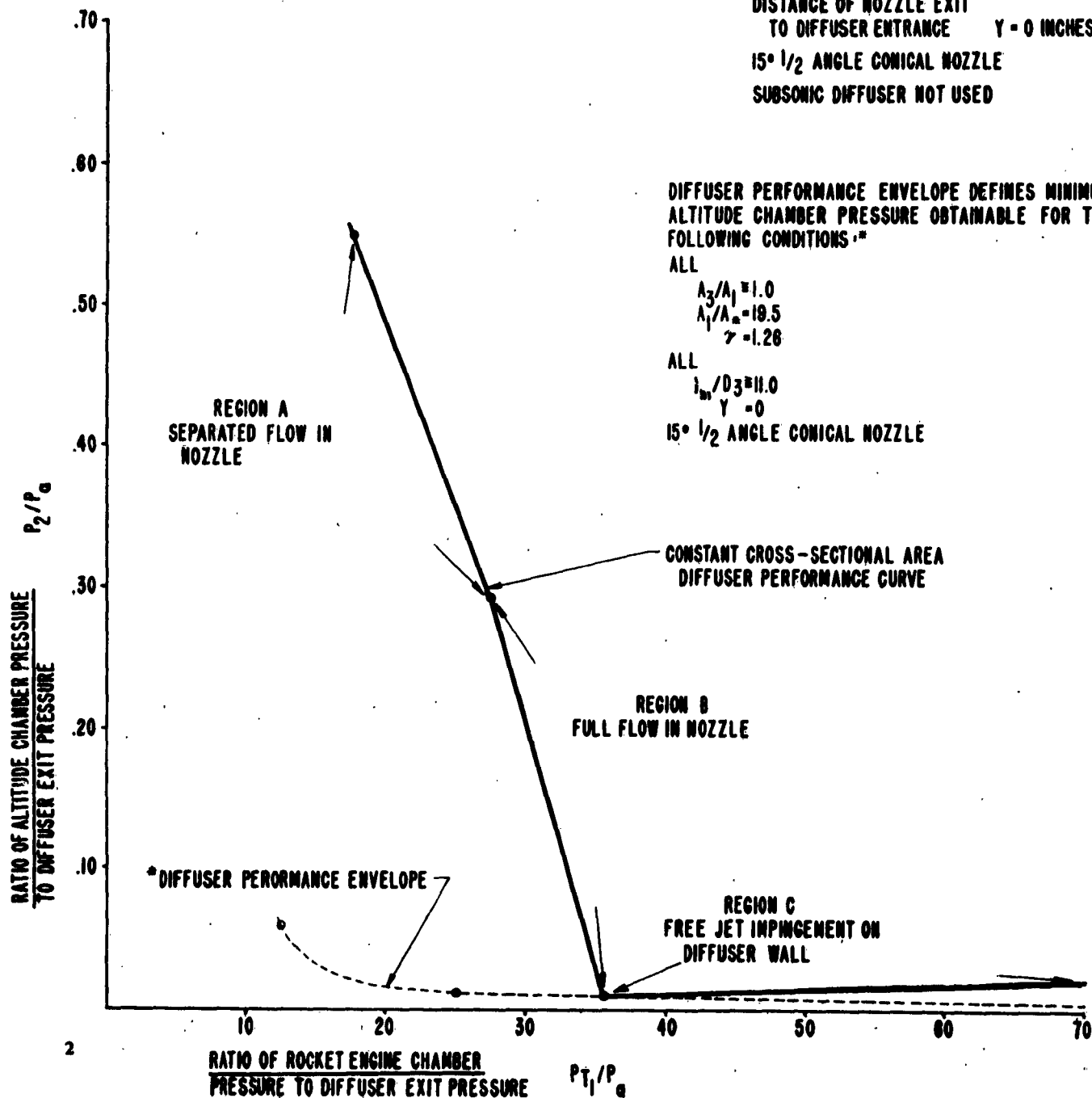
The diffuser development program was divided into two phases. Phase I testing was limited to straight cylindrical tubes while using nitrogen gas and later solid propellant rockets. Phase II testing was conducted with a two dimensional contoured diffuser with nitrogen gas as the primary fluid. Although Phase II testing was not comprehensive some advantages over the straight cylindrical tube configurations were indicated.

The results contained in this report represent an addendum to AFFTC-TR-60-1. Excerpts from the earlier report have been used to supplement this one.

The diffuser performance curve, referred to frequently in this report, is composed of three distinct types of diffuser flow; flow separation in the nozzle, full flow in the nozzle, and free jet impingement on the diffuser wall. The relation of these three types of diffuser flow are illustrated by regions A, B and C, respectively, of the diffuser performance curve on the next page.
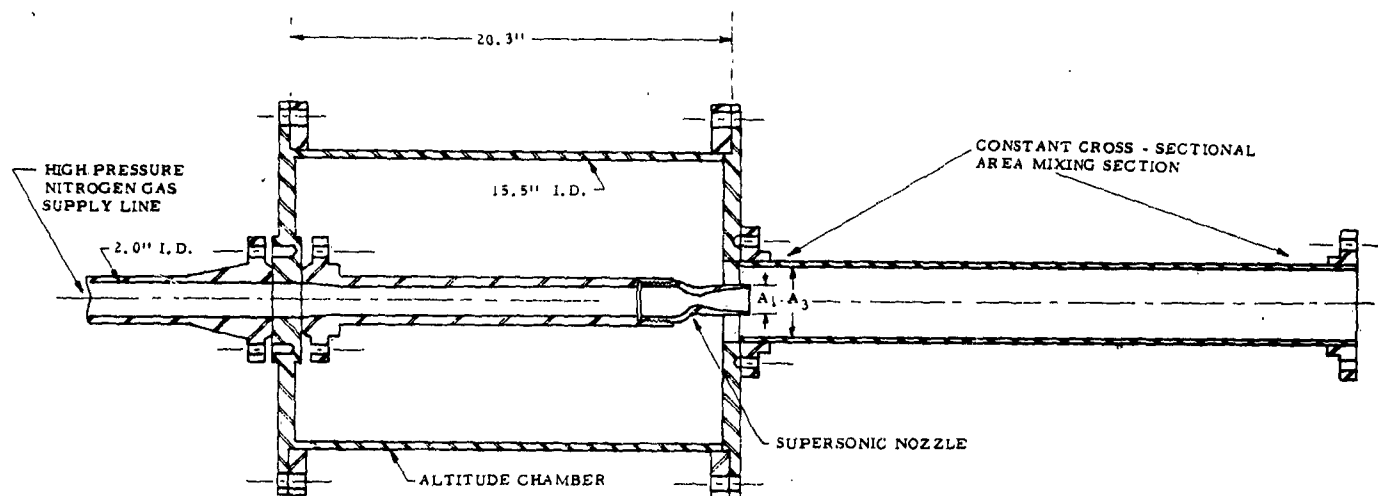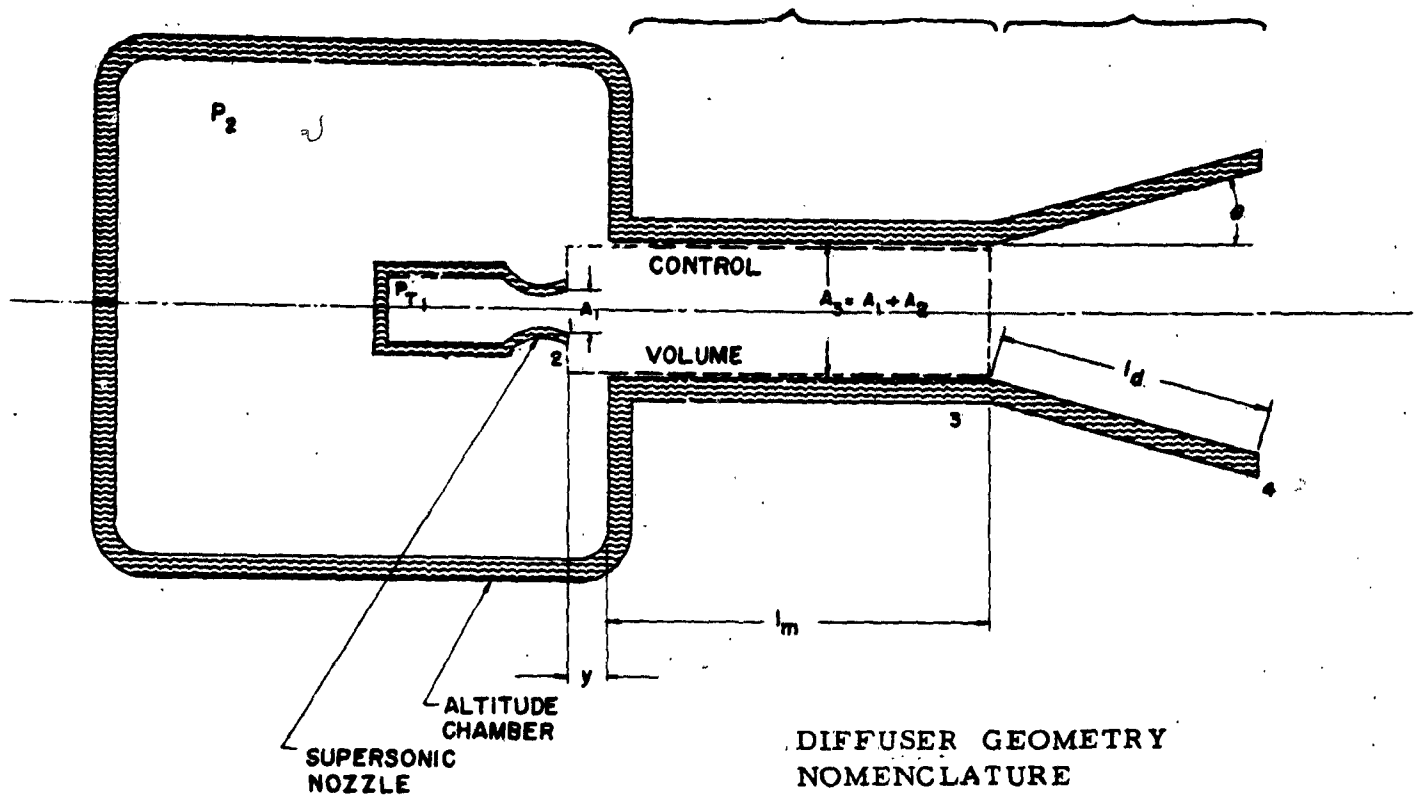
CHARACTERISTIC
DIFFUSER PERFORMANCE

RATIO OF SPECIFIC HEATS
OF EXHAUST GAS    $\gamma = 1.26$

$\dfrac{\text{AREA OF NOZZLE EXIT } (A_1)}{\text{AREA OF NOZZLE THROAT } (A_0)} = 19.5$

$\dfrac{\text{AREA OF DIFFUSER SECTION } (A_3)}{\text{AREA OF NOZZLE EXIT } (A_1)} = 2.7$

LENGTH OF DIFFUSER SECTION $(\ell_m) = 72$ INCHES

DISTANCE OF NOZZLE EXIT
TO DIFFUSER ENTRANCE    $Y = 0$ INCHES

$15°$ $1/2$ ANGLE CONICAL NOZZLE

SUBSONIC DIFFUSER NOT USED

DIFFUSER PERFORMANCE ENVELOPE DEFINES MINIMUM
ALTITUDE CHAMBER PRESSURE OBTAINABLE FOR THE
FOLLOWING CONDITIONS·*
ALL
$A_3/A_1 = 1.0$
$A_1/A_0 = 19.5$
$\gamma = 1.26$
ALL
$\ell_m/D_3 = 11.0$
$Y = 0$
$15°$ $1/2$ ANGLE CONICAL NOZZLE

REGION A
SEPARATED FLOW IN
NOZZLE

CONSTANT CROSS-SECTIONAL AREA
DIFFUSER PERFORMANCE CURVE

REGION B
FULL FLOW IN NOZZLE

*DIFFUSER PERORMANCE ENVELOPE

REGION C
FREE JET IMPINGEMENT ON
DIFFUSER WALL

RATIO OF ALTITUDE CHAMBER PRESSURE TO DIFFUSER EXIT PRESSURE    $P_2/P_a$

10    20    30    40    50    60    70

2

$\dfrac{\text{RATIO OF ROCKET ENGINE CHAMBER PRESSURE}}{\text{PRESSURE TO DIFFUSER EXIT PRESSURE}}$    $P_{T_1}/P_a$

CONSTANT CROSS-
SECTIONAL AREA        SUBSONIC DIFFUSER
MIXING SECTION

$P_2$

CONTROL

$P_{T_1}$     $A_1$     $A_3 = A_1 + A_2$

2     VOLUME     $l_d$

3

4

$l_m$

y

ALTITUDE
CHAMBER

SUPERSONIC
NOZZLE

DIFFUSER GEOMETRY
NOMENCLATURE

20.3"

HIGH PRESSURE
NITROGEN GAS
SUPPLY LINE

15.5" I.D.

CONSTANT CROSS - SECTIONAL
AREA MIXING SECTION

2.0" I.D.

$A_1 \cdot A_3$

SUPERSONIC NOZZLE

ALTITUDE CHAMBER

CONSTANT CROSS - SECTIONAL
AREA SUPERSONIC DIFFUSER

3

# DESCRIPTION OF APPARATUS

PHASE I

## Nitrogen Gas Test Apparatus:

The test apparatus consisted of an altitude chamber, supersonic nozzle and supersonic diffuser. The cylindrical altitude chamber, seen in the accompanying schematic, had an internal volume of approximately 2.2 cubic feet. During later tests this chamber was substituted for a larger chamber as a means of reducing evacuation time. The Mach 3, 4 and 5 supersonic nozzles were of electroformed nickel and had exit diameters $(D_1)$ ranging from 1.06 to 1.37 inches. A group of the nozzles used are seen in the accompanying photo. The supersonic diffusers were of stainless steel tubing having diameters $(D_3)$ ranging from 1.40 to 4.06 inches. A detailed list of nozzle and diffuser test articles are given in Table I.

High pressure nitrogen gas was regulated by a dome loaded pressure regulator to nozzle stagnation pressures varying from 10 psig to 1000 psig and was supplied at temperatures ranging from 0 degrees F to 50 degrees F. Unheated nitrogen gas was used in all tests.

Pressure sensing devices consisted of variable reluctance type transducers (0 to 20 psid, 0 to 500 and 0 to 1000 psig; Wiancko) and temperature compensated unbonded strain gage transducers (0 to 2 psia; Statham). Static pressures along the diffuser mixing section were calibrated from 0 to 15 psia and were indicated on oscillograph recorders. All other data was recorded on strip charts. Iron-constantan thermocouples were used to sense temperatures of the nitrogen gas supply and altitude chamber interior. Calibration of the absolute pressure transducers were accomplished at the test site with a precision mercury manometer, an electronic absolute pressure reference and a compensated barometer. System accuracy was within ±1 percent of full scale deflection during measurements of nozzle stagnation pressure and altitude chamber pressure while system accuracy for all other parameters was within ±2 percent of full scale recorder deflection.

4
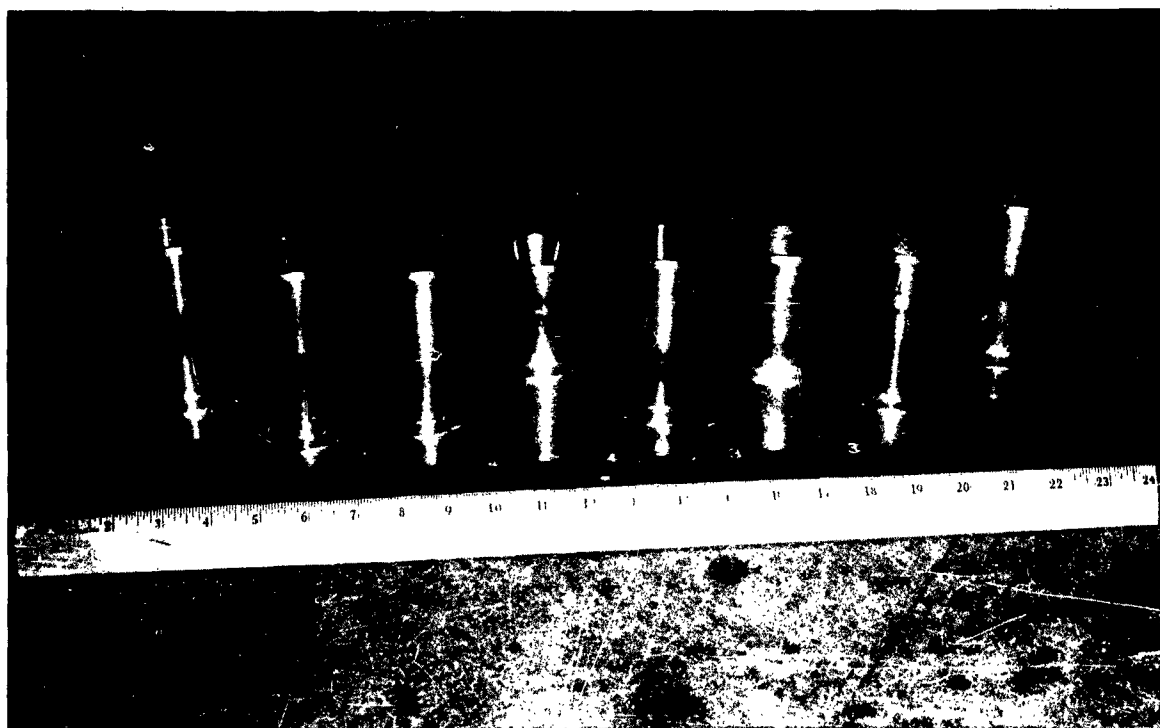
NITROGEN GAS SUPERSONIC NOZZLES

5

# TABLE I

## NITROGEN GAS TEST ARTICLES

| | Supersonic Nozzles | | Mixing Sections | | | |
|---|---|---|---|---|---|---|
| Mach No. | Throat Diameter-in | $D_1$ - in | $D_3$ - in | | $l_m/D_3$ | |
| 3 conical | .597 | 1.222 | 1.40 | 1.1 | 3.2 | 8.2 |
| 3 | .519 | 1.063 | 2.06 | 1.1 | 3.2 | 8.2 |
| 3 | .599 | 1.231 | 3.09 | 1.1 | 3.2 | 8.2 |
| 3 | .668 | 1.375 | 4.06 | 1.1 | 3.2 | 8.2 |
| 4 conical | .371 | 1.234 | | | | |
| 4 | .319 | 1.059 | | | | |
| 4. | .370 | 1.229 | | | | |
| 4 | .419 | 1.377 | | | | |
| 5 | .207 | 1.063 | | | | |
| 5 | .241 | 1.229 | | | | |
| 5 | .272 | 1.375 | | | | |

# TABLE II

## SOLID PROPELLANT TEST ARTICLES

| | 15 Degree Conical Supersonic Nozzles | | Mixing Sections | | |
|---|---|---|---|---|---|
| Mach No. | Throat Diameter - in | $D_1$ - in | $D_3$ - in | $l_m/D_3$ | $\theta$ - deg |
| 3.0 | .84 | 2.02 | 2.77 | 10.5 | 3.5 |
| 3.42 | .84 | 2.60 | 3.09 | 9.7 | 0 |
| 4.0 | .84 | 3.71 | 4.06 | 8.1 | 0 |
| 5.0 | .84 | 6.68 | 6.06 | 11.7 | 3.5 |
| | | | 7.50 | 16.0 | 3.5 |
| | | | 9.30 | 10.0 | 0 |
| | | | 10.00 | 10.0 | 0 |

-Solid Rocket Engine Test Apparatus:

The test apparatus consisted of 1000 pound thrust solid propellant rockets, Mach 3, 3.42, 4 and 5 fifteen degree conical supersonic nozzles, an altitude chamber having an internal volume of 578 cubic feet and supersonic diffuser sections with diameters $(D_3)$ varying from 2.77 to 10.0 inches. As shown below, standard JATO nozzles were modified for different Mach numbers. The solid rocket test apparatus is illustrated in the accompanying photographs. The supersonic diffuser sections all had length to diameter ratios $(l_m/D_3)$ in excess of 8. Temperatures along the diffuser wall were not recorded. Dimensions of the nozzles and mixing sections are given in Table II.

Instrumentation was the same as used in the nitrogen gas tests.
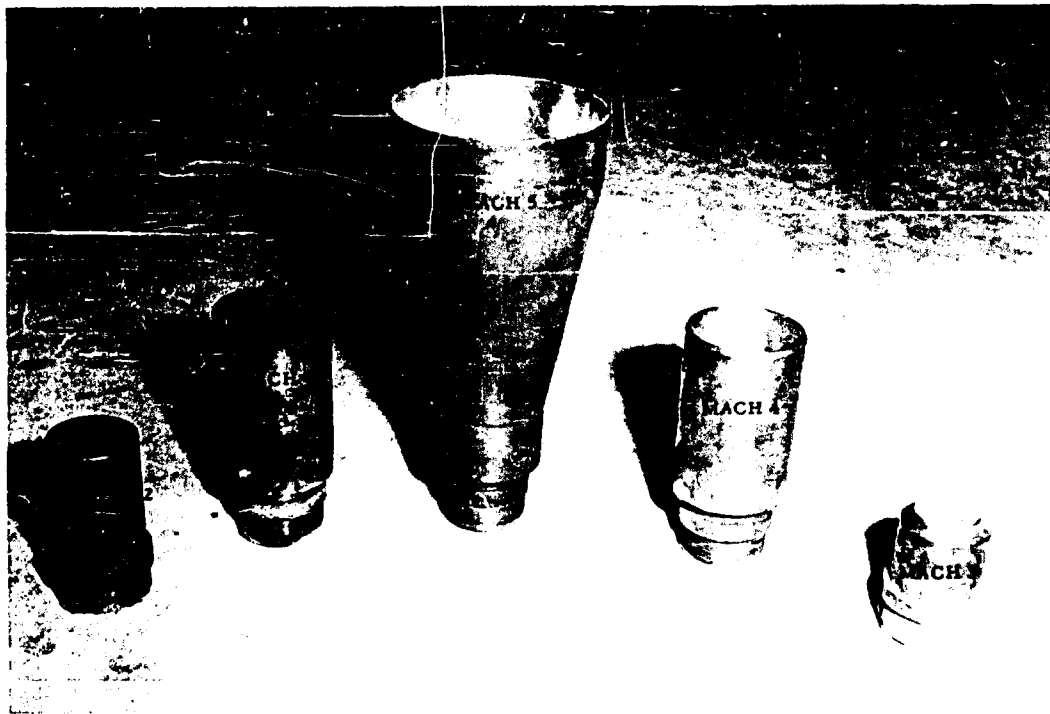


SOLID PROPELLANT NOZZLES

7

MACH 5 JATO NOZZLE

8
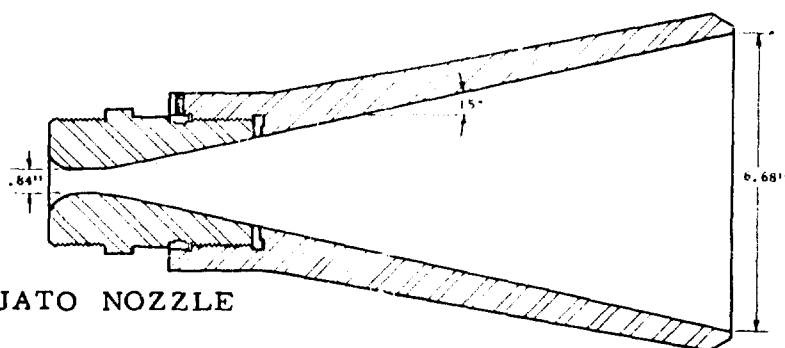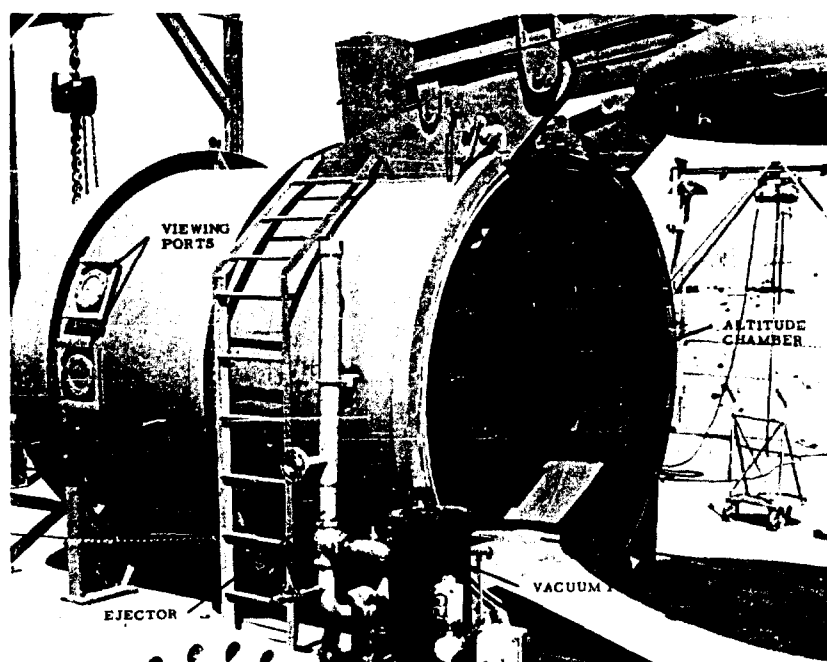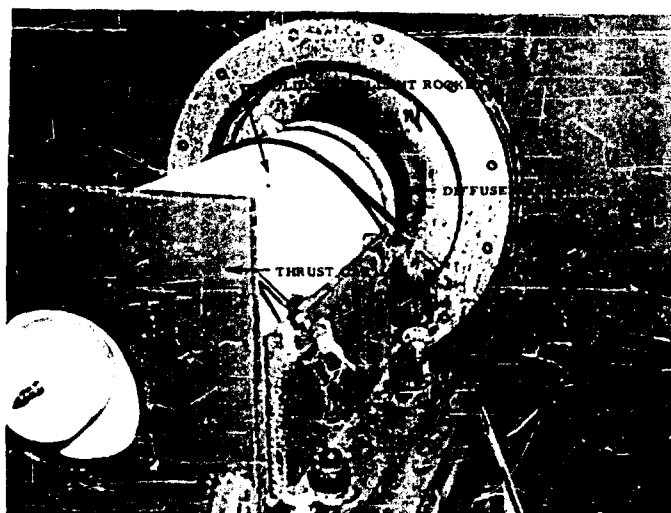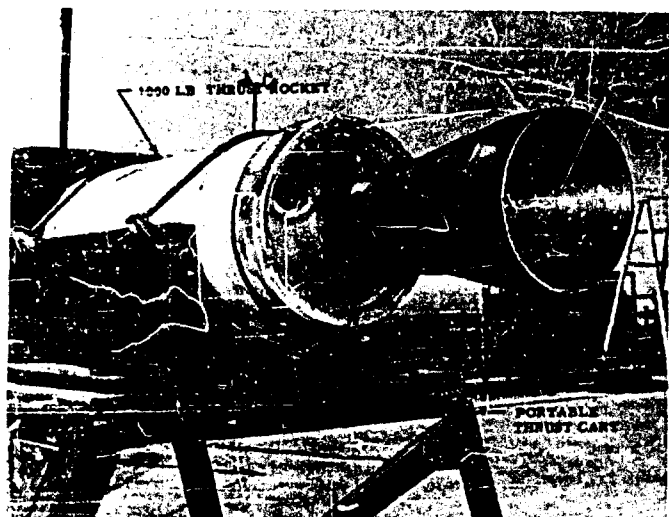
## PHASE II

### Nitrogen Gas Test Apparatus:

The nitrogen gas test apparatus as used during Phase II testing is shown in the accompanying schematic. The system is similar to the one used in Phase I nitrogen gas tests with the exception of the mixing section. The two dimensional mixing section is composed of removable contoured blocks. The blocks were adjustable during the tests by means of jack screws.

The instrumentation used in Phase II nitrogen gas tests was the same as that used in Phase I nitrogen gas tests with the exception of the static pressure recordings along the diffuser wall. These pressures were indicated on strip charts instead of recording oscillographs.



9
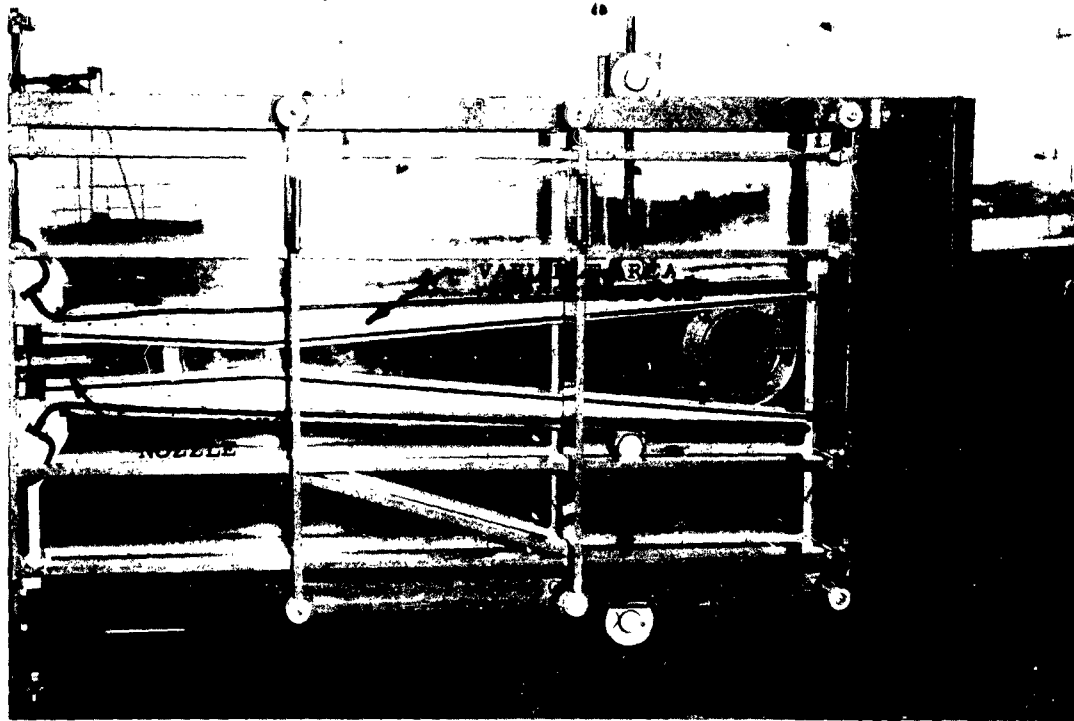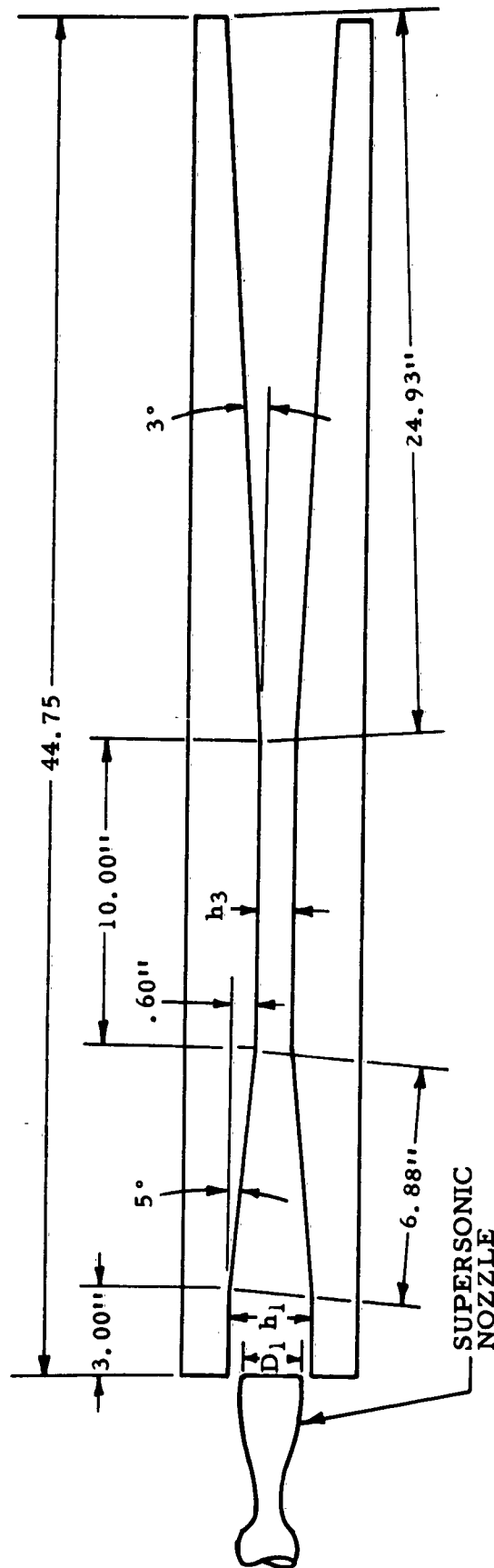
PHASE II TEST
ARTICLE SCHEMATIC

# TEST RESULTS

PHASE I

### Nitrogen Gas Results:

The Phase I tests were conducted with Mach 3, 4 and 5 supersonic nozzles and various cylindrical diffuser configurations (Table I). Bell and conically shaped nozzles were used in the Mach 3 and 4 tests while only bell shaped nozzles were available for the Mach 5 tests. The ratio of diffuser mixing length to diffuser diameter $(l_m/D_3)$ ranged between 8 and 11 in the majority of tests while a few tests were conducted with ratios of less than 8.

The results appearing in Appendix I indicate tests with diffuser configurations of mixing length to diffuser diameter $(l_m/D_3)$ ratios greater than 8 had steady state experimental points which always fell very close to or just to the right of the analytical line in region B of the diffuser performance curve (page 2). A marked deviation from the analytical line was experienced when diffuser configurations having mixing length to diffuser diameter $(l_m/D_3)$ ratios of less than 8 were tested. A performance curve illustrating this effect is shown in Fig. 22, Appendix I. It is of interest to note that regardless of mixing length to diffuser diameter $(l_m/D_3)$ ratio the linear portion of the diffuser performance curve, region C, always extrapolated through the origin.

The test results of the program have verified prior assumptions of the principle factors governing diffuser performance. The principle factors being area ratio of diffuser area to nozzle exit area $(A_3/A_1)$, nozzle Mach number (M), nozzle shape, total nozzle pressure and specific heat ratio of the exhaust gas.

During the tests, the position of the nozzle was maintained at the plane of the mixing section entrance or just inside. A few tests were run with the nozzle just outside of the mixing section entrance; i.e., y equal to a positive increment. It was noted that diffuser performance suffered to an extent. Further testing was not conducted with this configuration because it was desirable to have the jet exiting from the nozzle impinge on the inside diameter of the mixing section or supersonic diffuser wall.

Altitude chamber pressure $(P2_0)$ was at ambient conditions immediately prior to the majority of tests. A few tests were accomplished with small initial values of altitude chamber pressure $(P2_0)$ just prior to the test. The objective of these particular tests was to simulate diffuser "starting" at altitude and to determine if hysteresis of the diffuser performance curve existed. Hysteresis was not noted in diffuser performance provided the diffuser section was of adequate length.

11

## Results of Solid Propellant Tests:

Tests were conducted with 21 diffuser configurations during the solid propellant tests. The rocket motor used during all testing was the 14-DS-1000 JATO unit. The nozzle for this unit was modified to permit testing at Mach numbers of 3, 3.42, 4 and 5.

One set of data points for each test was taken under steady state conditions and plotted as functional values of the altitude chamber pressure for a particular rocket motor chamber pressure. The slope of this line in region C was determined by a straight line through the steady state data point and the origin. The intersection of this line with the analytical solution of region B established the cut-off point or "knee" of the diffuser performance curve. The envelopes in Figs. 10, 11, 12 and 13, Appendix I, define the minimum altitude chamber pressures obtainable for a family of diffuser performance curves.

The results of these tests concur with the results obtained during the nitrogen gas tests; i.e., Mach number, area ratio and specific heat ratio are the principle factors governing diffuser performance. Only conical nozzles were used and therefore, the affect of nozzle shape upon diffuser performance could not be determined during this series of tests.

Solid propellant motor tests were conducted with initial altitude chamber pressures at ambient conditions.


## PHASE II

### Results of Two-Dimensional Variable Area Tests:

A series of tests using a variable area two-dimensional contoured diffuser was conducted at AFFTC by Sandberg-Serrell Corporation personnel. The diffuser is shown in the accompanying photograph. Mach 3, 4 and 5 nozzles having exit diameters of 1.229 inches were used in conjunction with the diffuser blocks as shown on page 9.

A series of tests with each nozzle was conducted to determine "starting" conditions and conditions necessary to obtain maximum altitude simulation. Diffuser performance envelopes, analogous to constant area diffuser performance envelopes, are presented in Fig. 25, Appendix I, for comparison purposes.

12

# DESIGN CONSIDERATIONS

The test results of this program have verified prior assumptions governing diffuser performance in region B. The test results, as indicated earlier, have shown the principle factors in determining region B are: Mach number; specific heat ratio of the exhaust gases; nozzle shape; and area ratio of diffuser area to nozzle exit area $(A_3/A_1)$. The one dimensional equations (Appendix I) assume however, that adequate mixing length $(l_m)$ exists. It has been shown in the results, (Fig. 22, Appendix I) that diffuser performance can be extensively altered. An indication of necessary mixing length to diffuser diameter $(l_m/D_3)$ ratio as a function of Mach number has been included in Fig. 21, Appendix I. One dimensional equations agree quite well with experimental results of diffuser performance in region B to the point where increasing $PT_1$ produces choking of secondary flow in the mixing section. At this point a transition occurs to region C and in this region diffuser performance can no longer be predicted by elementary treatments. A plot of region C slopes, altitude pressure to nozzle total pressure $(P_2/PT_1)_c$ as a function of area ratio $(A_3/A_1)$, Mach number, nozzle shape, and specific heat ratio ( $\gamma$ ) is shown in Appendix I (Figs. 14 through 20).

The results have shown a marked performance difference in region C as related to nozzle shape when all other geometric considerations are constant. The performance curves obtained with the bell shaped nozzle always show a lower slope in region C, hence, improved performance while region B appears to remain constant regardless of nozzle shape.

Consideration of nozzle shapes indicate that the flow angle at the nozzle exit must be greater, in the case of bell nozzle, to cause impingement on the diffuser wall since the flow just inside the bell nozzle is parallel to the nozzle axis. The larger angle necessary for impingement is produced at the expense of increasing $PT_1$ in region B and thereby causes a decrease in the value of $P_2$.

The effect of specific heat ratio ( $\gamma$ ) in region B can be readily determined from the one dimensional equations describing diffuser flow. The effect of specific heat ratio ( $\gamma$ ) in region C has been shown in the slope plots that for values of decreasing specific heat ratio ( $\gamma$ ) the slope of region C diminishes when considering all other parameters constant so that in fact better altitude simulation is possible at the expense of increased primary stagnation pressure. Information from this and other reports, mentioned in the Reference section, Appendix I, should be of assistance in empirically predicting diffuser performance in region C.

13

As an illustration of the above discussion, an example for designing a diffuser is presented.

Given:

Specific heat ratio of exhaust gases, $(\gamma)$   = 1.26

15 degree conical nozzle, Mach number   = 4.00

Combustion chamber pressure, $P_{T_1}$   = 400 psia

Nozzle exit area, $A_1$   = 86.6 sq in.

Corrected barometric reading, $P_a$   = 13.20 psia

Find:

Diffuser area to nozzle exit area $(A_3/A_1)$

and

Altitude pressure to atmospheric pressure $(P_2/P_a)_{opt}$

Solution: Referring to the performance envelope, Fig. 12, Appendix I, the ordinate at $P_{T_1}/P_a = 30.3$ intersects at $P_2/P_a = .010$. The slope of region C is determined since the linear portion of the performance curve extrapolates through the origin. In this case the slope $P_2/P_{T_1} = .010/30.3 = .00033$. Referring to the corresponding slope plot (Fig. 19, Appendix I) the area ratio is found to be 2.2. It is desirable to operate in region C to the right of the cut-off point or "knee" of the performance curve so that in practice a slightly smaller area ratio $(A_3/A_1)$ is chosen to insure operation in region C. In this case an area ratio of 2.0 is arbitrarily chosen. Therefore, $A_3 = 2.0 \times 86.6 = 173.2$ inches$^2$. Referring to Fig. 19, Appendix I, it is seen that the slope of region C at an area ratio of 2.0 = .00039. The desired $A_3/A_1 = 2.0$ and $P_2/P_{T_1} = .00039$. Then $P_2/P_a = .0118$ which is equivalent to an altitude of approximately 100,000 feet.

14

# APPENDIX I

## REFERENCES

1. Adamson, T.C.,Jr., Nicholls, J.A., "On the Structure of Jets from Highly Under Expanded Nozzles into Still Air." Journal of the Aero/Space Sciences, Vol 26, No. 1, January 1959.

2. Barton, D. L. and Taylor, D., "An Investigation of Ejectors Without Induced Flow." Phase I AEDC-TN-59-145; December 1959.

3. Crocco, L., "One Dimensional Treatment of Gas Dynamics." Part B of Fundamentals of Gas Dynamics, Emmons, H. W., Ed. Princeton University Press, Inc., New York, 1958.

4. De Leo, R. V., and Rose, R.E., "An Experimental Investigation of a Special Air Ejector for the Ram Jet Addition Exhauster System of AEDC." AEDC-TR-55-46, March 1955.

5. Dryden, H. L., and von Karman, Th., Ed., "Advances in Applied Mechanics, Vol V, Ch. 1. "Supersonic Air Ejectors." Academic Press, Inc., New York, N. Y., 1958.

6. Englert, G.W., "Operational Method of Determining Initial Contour of and Pressure Field about a Supersonic Jet." NASA TND-279; April 1960.

7. Fabri, J. and Pavlon, J., "Theory and Experiments on Supersonic Air-to-Air Ejectors." NACA IM No. 1410, September 1958.

8. Ferri, A., "The Method of Characteristics." Section G of General Theory of High Speed Aerodynamics, Sears, W.R., ED. Princeton University Press, Princeton, N.Y., 1954.

9. Fortini, A., "Performance Investigation of a Nonpumping Rocket-Ejector System for Altitude Simulation." NASA TND-257; December 1959.

10. Foster, R. M., "The Supersonic Diffuser and its Application to Altitude Testing of Captive Rocket Engines." AFFTC-TR-60-1, January 1960.

15

11.  Janes, W. L., Price, H.G., Jr., Lorenzo, C.F. "Experimental
     Study of Zero-Flow Ejectors using Gaseous Nitrogen." NASA
     TND-203, March 1960.

12.  Kepler, C.E., "Investigation of the Performance of a Variable -
     Geometry Diffuser for the AEDC Propulsion Wind Tunnel." 
     AEDC TR-55-52, August 1955.

13.  Latvala, E.K., "Spreading of Rocket Exhaust Jets at High
     Altitude." AEDC-TR-59-11, June 1959.

14.  Love, E.S., and Grigsby, E.E., "Some Studies of Axisymmetric
     Free Jets Exhausting from Sonic and Supersonic Nozzles into Still
     Air and into Supersonic Streams." NACA RML54L31, May 1955.

15.  Love, E.S. and Lee, L.P., "Shape of Initial Portion of
     Boundary of Supersonic Axisymmetric Free Jets at Large Jet
     Pressure Ratios." NACA TN 4195, January 1958.

16.  Moe, M.M. and Troesch, B.A., "The Computation of Jet Flows
     with Shocks." Space Technology Labs, Inc. TR-59-0000-00661,
     1959.

17.  Rhodes, R.P., "Evaluation of Hot-Water-Driven Diffuser Ejectors."
     AEDC TN-59-127, November 1959.

18.  Schlumpf, R.L. and Clark, H. K., "Spreading of Hydrogen Air
     Rocket Exhaust Jets at High Altitudes." AEDC-TN-59-131,
     November 1959.

19.  Shapiro, A.H., "The Dynamics and Thermodynamics of
     Compressible Fluid Flow." Vol I, Ronald Press Co., New York
     1953.

20.  Wang, C.J., Peterson, J.B., and Anderson, R., "Gas Flow
     Tables." Space Technology Laboratories, Inc., GM-TR-154,
     14 March 1957.

21.  Wilsted, H.D., Huddleston, S.D., and Ellis, C. W., "Effect of
     Temperature on Performance of Several Ejector Configurations."
     NACA No. E9E16, no date.

16

# NOMENCLATURE

| | |
|---|---|
| A | cross-sectional area |
| $C_f$ | dimensionless friction factor ( $\frac{\tau_w}{1/2\rho V^2}$ ) |
| $C_p$ | specific heat at constant pressure |
| $C_v$ | specific heat at constant volume |
| D | diameter, inches |
| F | force |
| l | length, inches |
| M | Mach number |
| P | static pressure, absolute |
| $P_T$ | stagnation or total pressure, absolute |
| y | axial distance along diffuser from the constant area mixing section inlet to the nozzle exit |
| $\gamma$ | ratio of specific heats, $C_p/C_v$ |
| $\theta$ | half angle of subsonic diffuser outlet |
| $\rho$ | mass density |
| $\tau_w$ | friction force per unit area at the wall |

Subscripts

| | |
|---|---|
| 1 | primary flow conditions (flow conditions in the exit of the supersonic nozzle) |
| 2 | secondary flow conditions or altitude chamber conditions |
| 3 | conditions in the diffuser mixing section |
| 4 | conditions in the subsonic diffuser exit |
| a | atmospheric or ambient conditions |
| c | region C of the performance curve (see page 2 ) |
| d | subsonic diffuser outlet |
| m | constant cross-sectional area mixing section |
| o | initial conditions |
| T | stagnation or total state |
| x | direction along diffuser wall |
| bell | bell shaped nozzle |
| con | conical nozzle |

## DIFFUSER FLOW EQUATIONS

### Continuity Equation:

$$\frac{M_3^*}{\left(1 - \dfrac{\gamma_1 - 1}{\gamma_1 + 1} M_3^{*2}\right)} = \frac{P_{T_1} A_1 M_1^*}{P_3 A_3} \left(1 - \frac{\gamma_1 - 1}{\gamma_1 + 1} M_1^{*2}\right)^{\frac{1}{\gamma_1 - 1}}$$

First law of thermodynamics

$$T_{T_1} = T_{T_3}$$

### Momentum Equation:

$$\frac{P_{T_1} A_1}{P_3 A_3} \left(1 - \frac{\gamma_1 - 1}{\gamma_1 + 1} M_1^{*2}\right)^{\frac{1}{\gamma_1 - 1}} (1 + M_1^{*2}) + \frac{P_2 A_2}{P_3 A_3} = \frac{1 + M_3^{*2}}{1 - \dfrac{\gamma_1 - 1}{\gamma_1 + 1} M_3^{*2}}$$

$$+ \frac{F_x}{P_3 A_3}$$

where

$$\frac{F_x}{P_3 A_3} = 4C_f \frac{l_m}{D_3} \frac{\gamma_1}{\gamma_1 + 1} \frac{M_3^{*2}}{1 - \dfrac{\gamma_1 - 1}{\gamma_1 + 1} M_3^{*2}}$$

Figure 1    Diffuser Design Curve

**MACH 3 SLOWLY DIVERGENT BELL NOZZLE-DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDER DIFFUSERS
WITH AND WITHOUT SUBSONIC DIFFUSERS**
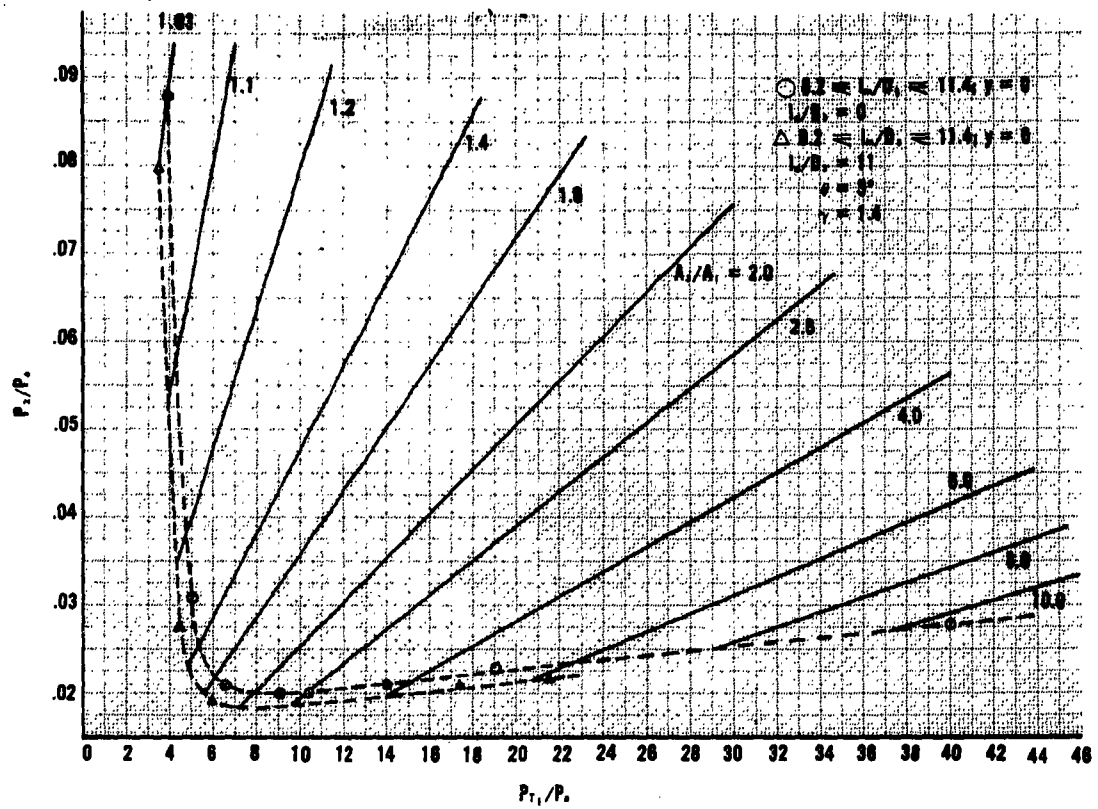


Figure 2    Diffuser Design Curve

**MACH 3 CONICAL 15° NOZZLE-DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDER DIFFUSERS
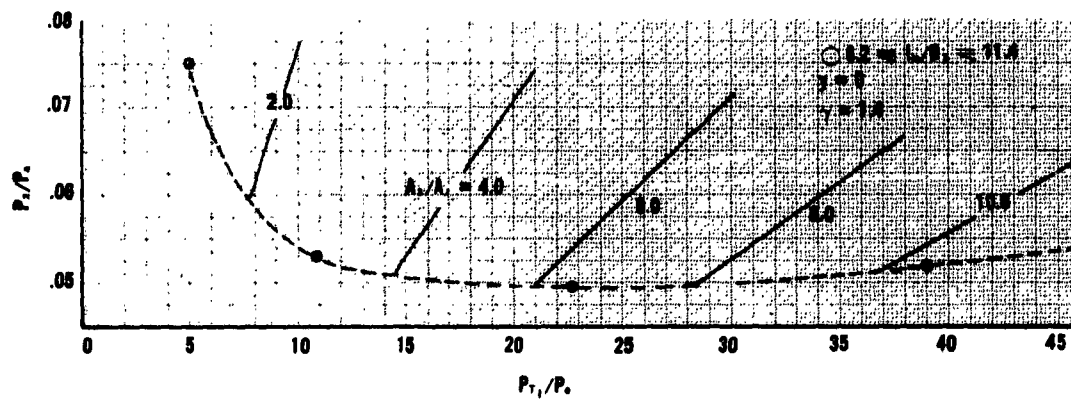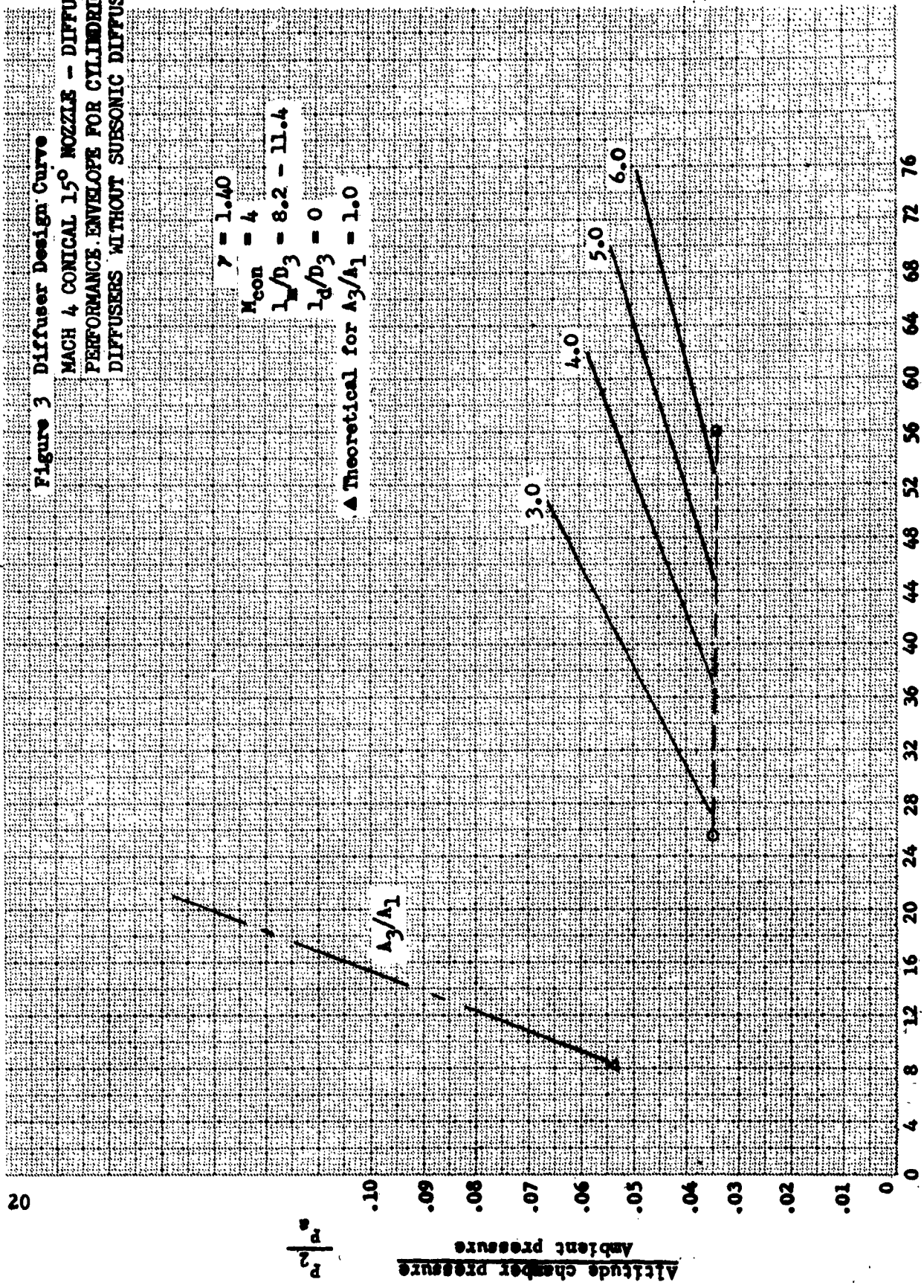WITHOUT SUBSONIC DIFFUSERS**



19

Figure 3  Diffuser Design Curve
MACH 4 CONICAL 15° NOZZLE – DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDRICAL
DIFFUSERS WITHOUT SUBSONIC DIFFUSERS

$\gamma = 1.40$
$M_{con} = 4$
$l_m/D_3 = 8.2 - 11.4$
$l_d/D_3 = 0$

▲ Theoretical for $A_3/A_1 = 1.0$

$A_3/A_1$

$\dfrac{P_{t_1}}{P_a}$ $\quad$ Total nozzle pressure / Ambient pressure

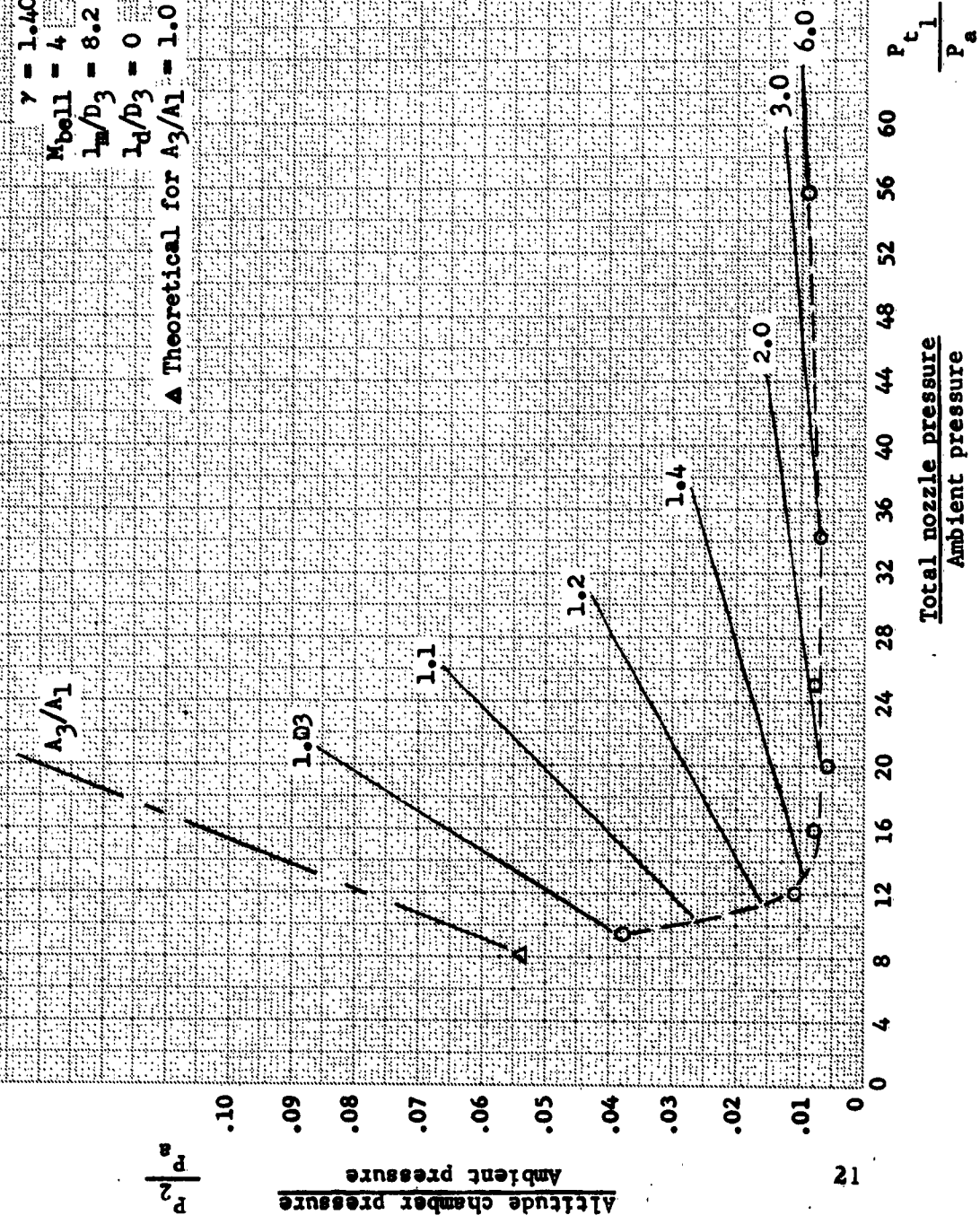$\dfrac{P_2}{P_a}$ $\quad$ Altitude chamber pressure / Ambient pressure

Figure 4  Diffuser Design Curve

MACH 4 SLOWLY DIVERGENT BELL NOZZLE - DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS
WITHOUT SUBSONIC DIFFUSERS

$\gamma$ = 1.40
$M_{bell}$ = 4
$l_m/D_3$ = 8.2 - 11.4
$l_d/D_3$ = 0

△ Theoretical for $A_3/A_1$ = 1.0

$A_3/A_1$

1.0D3

1.1

1.2

1.4

2.0

3.0

6.0

$\dfrac{P_{t_1}}{P_a}$

Total nozzle pressure
Ambient pressure

$\dfrac{P_a}{P_2}$

Altitude chamber pressure
Ambient pressure

.10  .09  .08  .07  .06  .05  .04  .03  .02  .01  0

0   4   8   12  16  20  24  28  32  36  40  44  48  52  56  60

21

Figure 5  Diffuser Design Curve

MACH 5 SLOWLY DIVERGENT BELL NOZZLE – DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS
WITHOUT SUBSONIC DIFFUSERS

$\gamma = 1.40$
$M_{bell} = 5$
$L_n/D_3 = 8.2 - 11.4$
$L_d/D_3 = 0$

▲ Theoretical for $A_3/A_1 = 1.0$
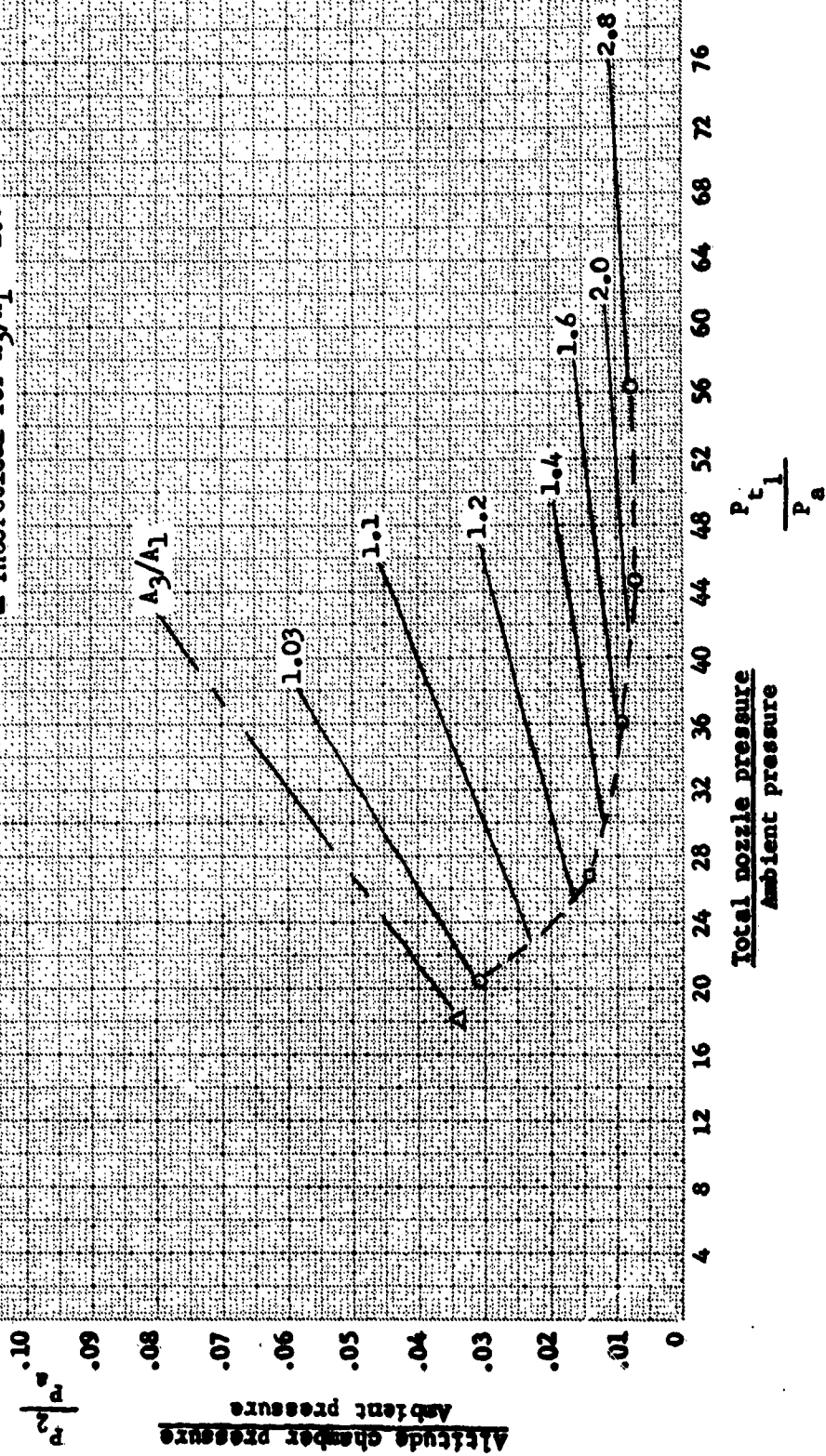
Figure 6

ANALYTICAL SOLUTION OF THE ONE – DIMENSIONAL STEADY
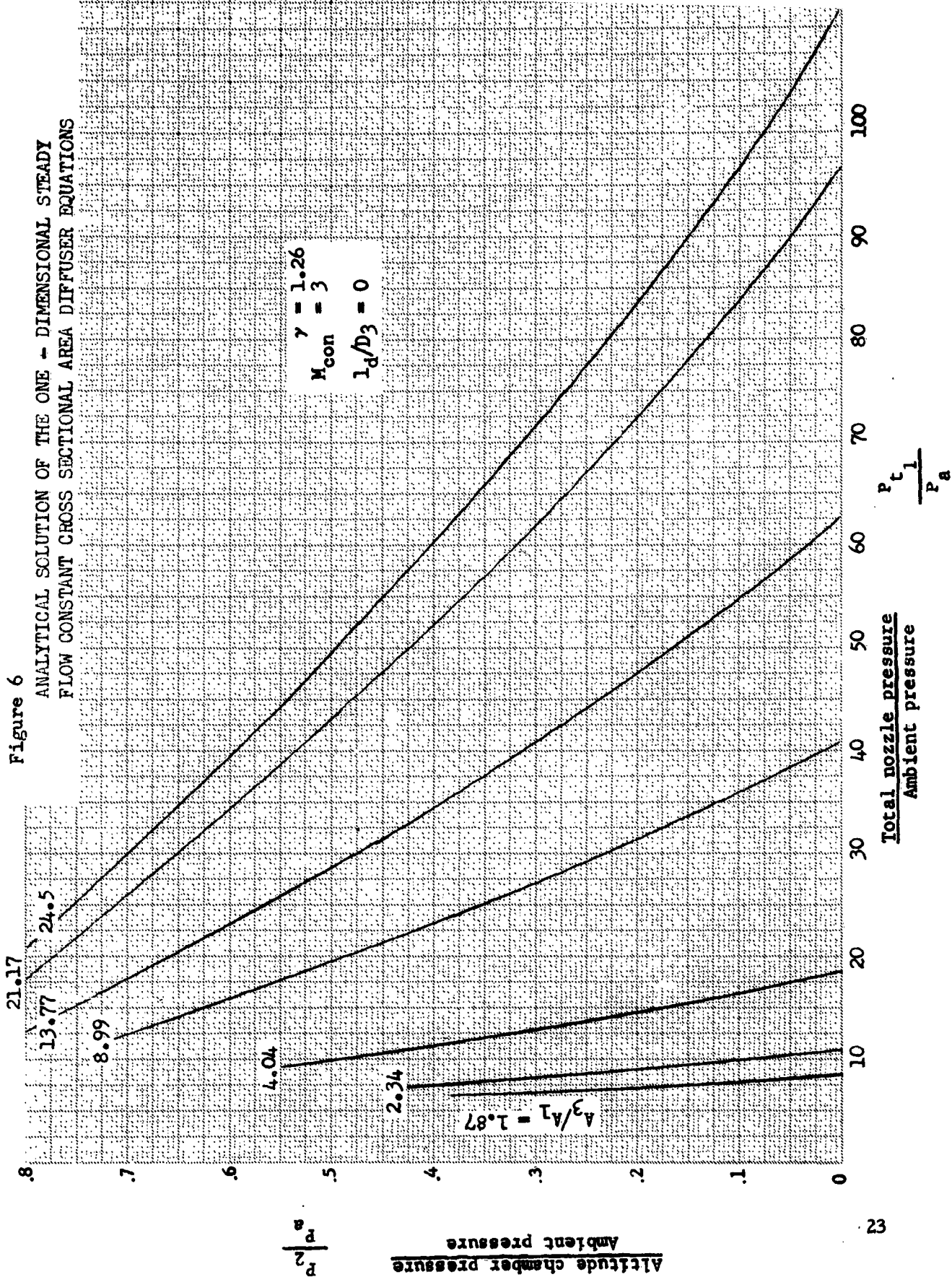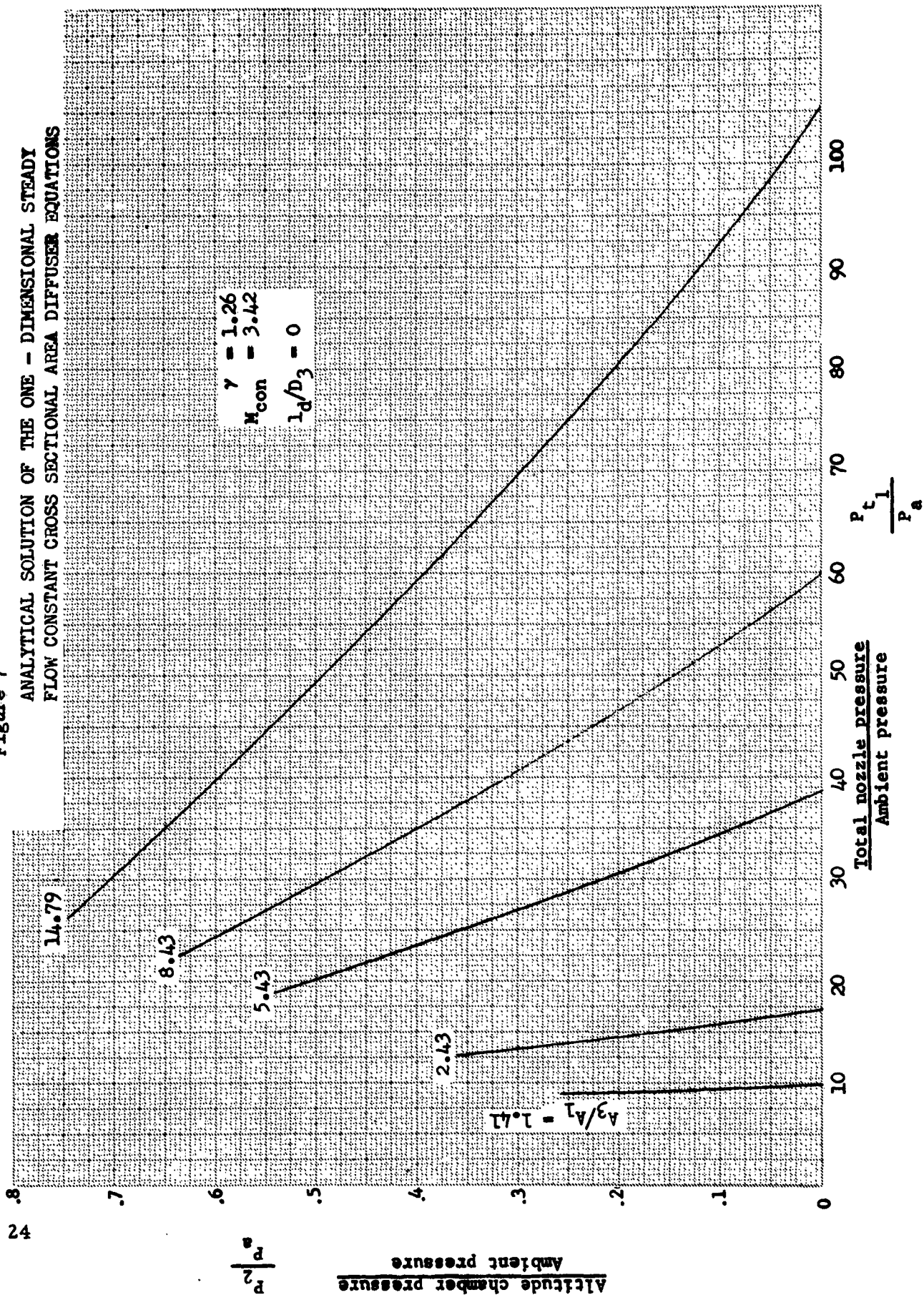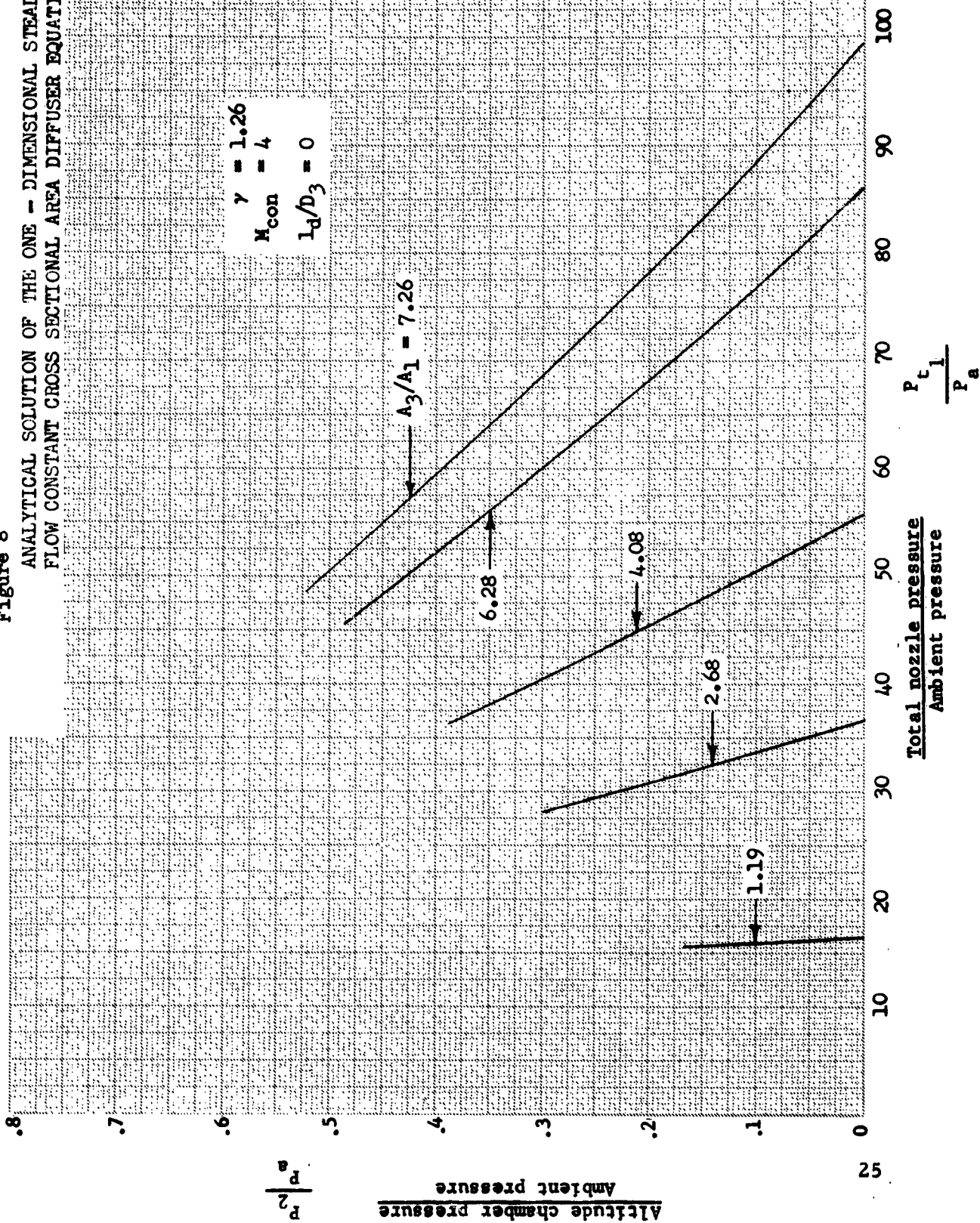FLOW CONSTANT CROSS SECTIONAL AREA DIFFUSER EQUATIONS

$\gamma = 1.26$
$M_{con} = 3$
$l_d/D_3 = 0$

$\dfrac{P_{t_1}}{P_a}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$

$\dfrac{P_2}{P_a}$

$\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

$A_3/A_1 = 1.87$

24.5
21.17
13.77
8.99
4.04
2.34

23

Figure 7

ANALYTICAL SOLUTION OF THE ONE – DIMENSIONAL STEADY
FLOW CONSTANT CROSS SECTIONAL AREA DIFFUSER EQUATIONS

$\gamma = 1.26$
$M_{con} = 3.42$
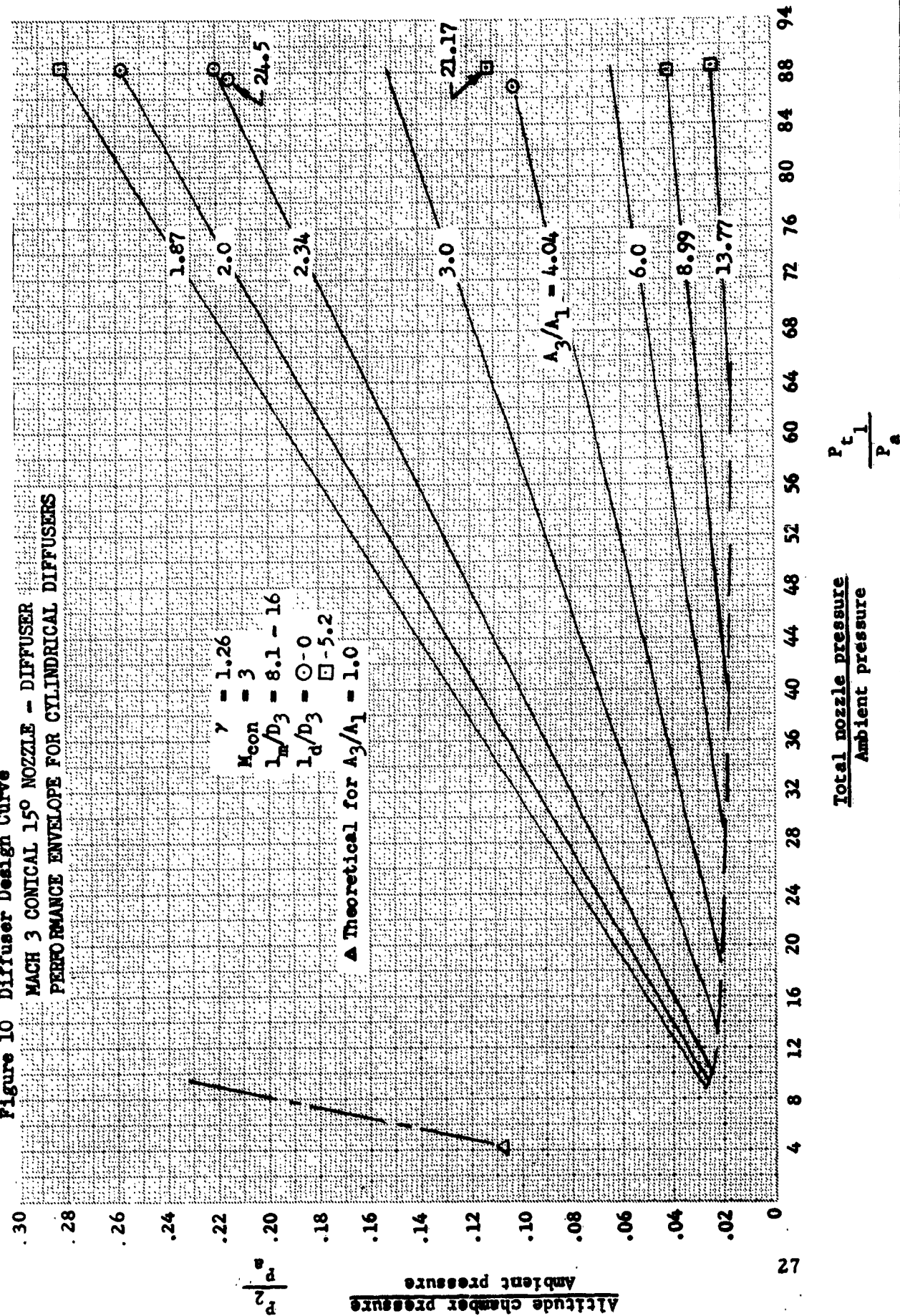$l_d/D_3 = 0$

$\dfrac{P_{t_1}}{P_a}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$

$\dfrac{P_2}{P_a}$

$\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

14.79

8.43

5.43

2.43

$A_3/A_1 = 1.41$

24

Figure 8

ANALYTICAL SOLUTION OF THE ONE – DIMENSIONAL STEADY
FLOW CONSTANT CROSS SECTIONAL AREA DIFFUSER EQUATIONS

$\gamma = 1.26$
$M_{con} = 4$
$l_d/D_3 = 0$

$A_3/A_1 = 7.26$

6.28

4.08

2.68

1.19

$\dfrac{P_{t_1}}{P_a}$

Total nozzle pressure
Ambient pressure

$\dfrac{P_2}{P_a}$

Altitude chamber pressure
Ambient pressure

25

Figure 9

ANALYTICAL SOLUTION OF THE ONE — DIMENSIONAL STEADY
FLOW CONSTANT CROSS SECTIONAL AREA DIFFUSER EQUATIONS



26
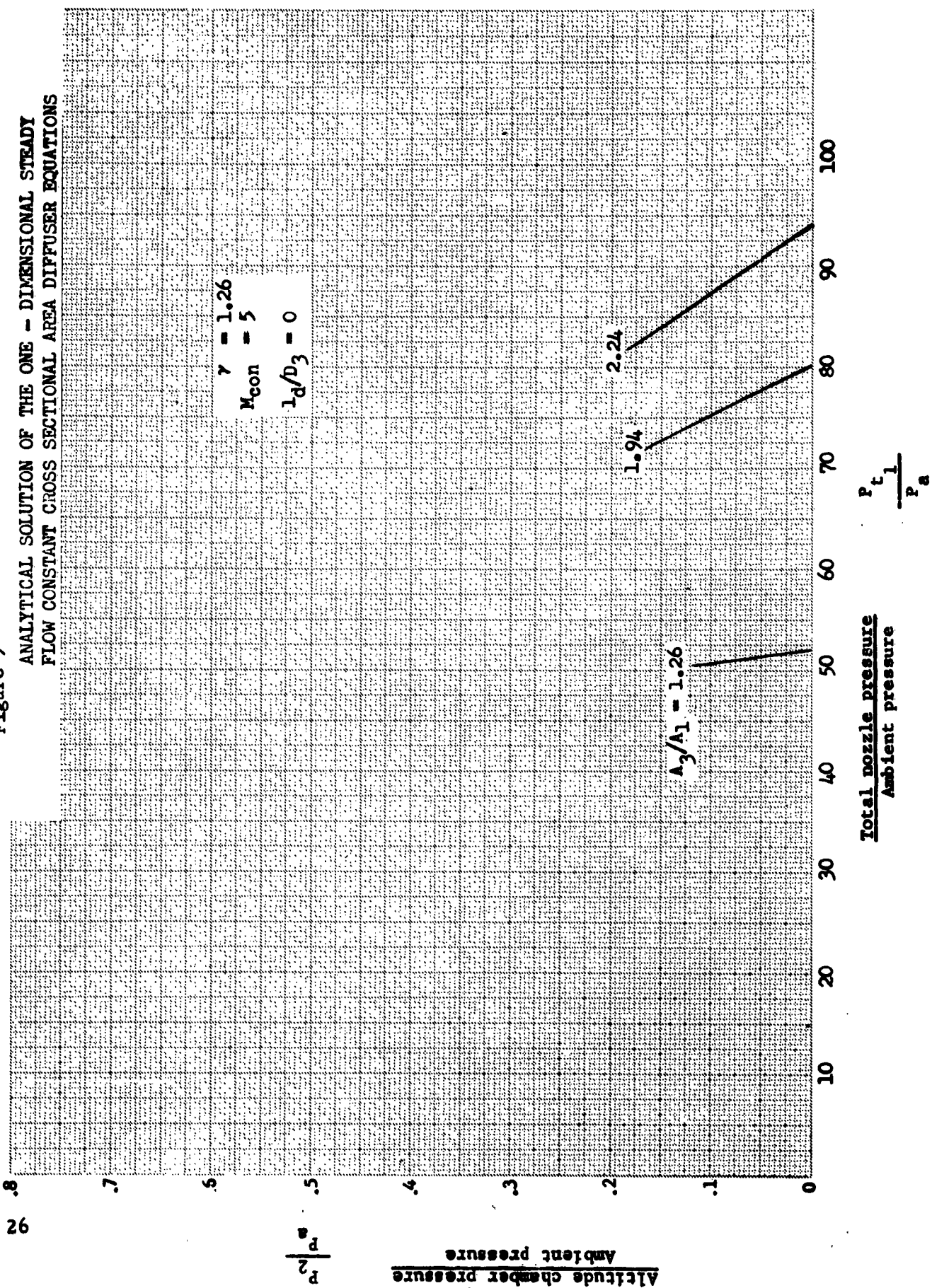
Figure 10 Diffuser Design Curve

MACH 3 CONICAL 15° NOZZLE – DIFFUSER
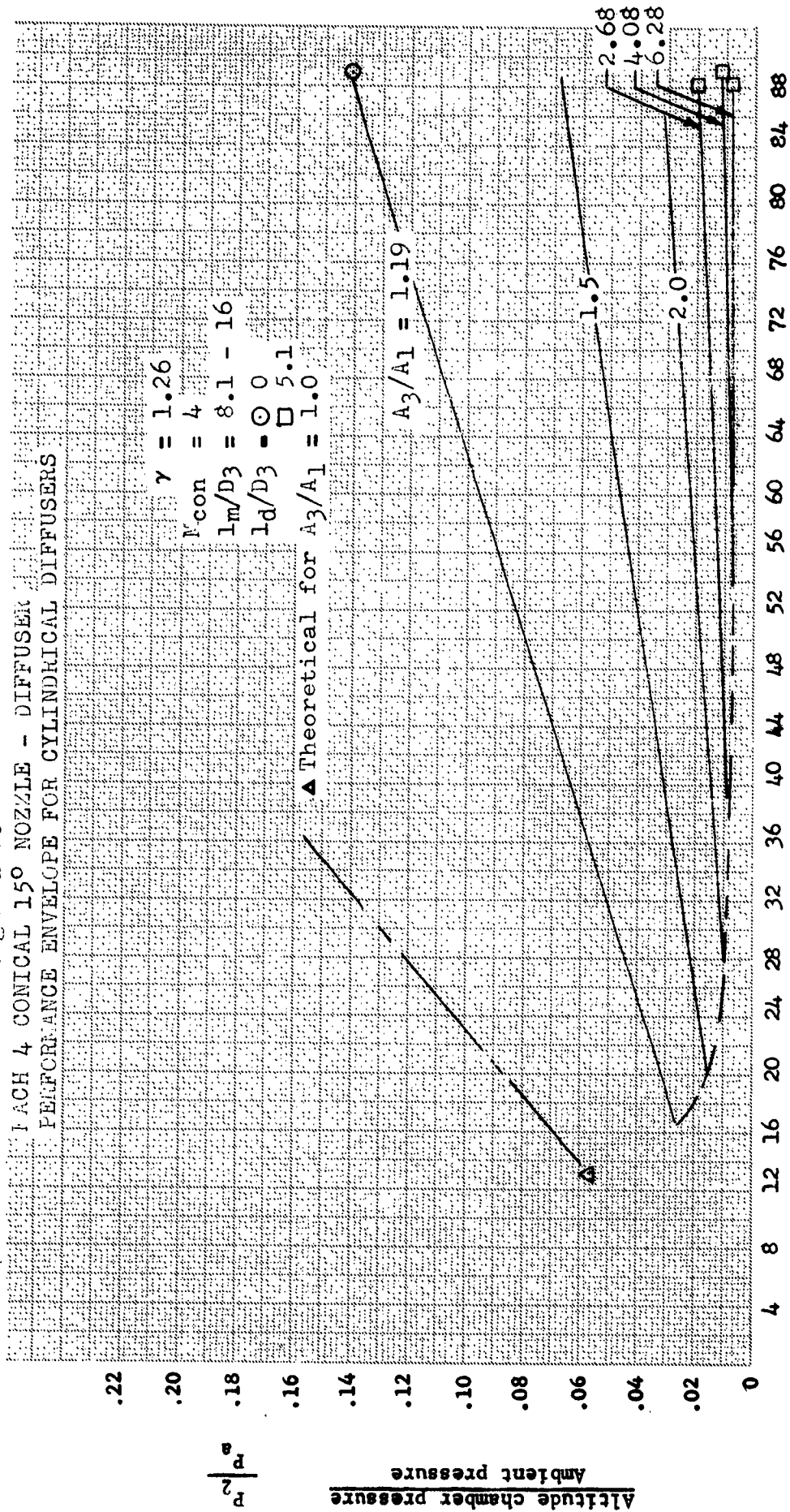PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS

$\gamma = 1.26$
$M_{con} = 3$
$l_m/D_3 = 8.1 - 16$
$l_d/D_3 = \odot - 0$
$\boxdot - 5.2$

▲ Theoretical for $A_3/A_1 = 1.0$

$A_3/A_1 = 4.04$

1.87
2.0
2.34
24.5
3.0
21.17
6.0
8.99
13.77

$\dfrac{P_2}{P_a}$

Altitude chamber pressure
Ambient pressure

$\dfrac{P_{t_1}}{P_a}$

Total nozzle pressure
Ambient pressure

27

Figure 11 Diffuser Design Curve

MACH 3.42 CONICAL 15° NOZZLE – DIFFUSER

PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS
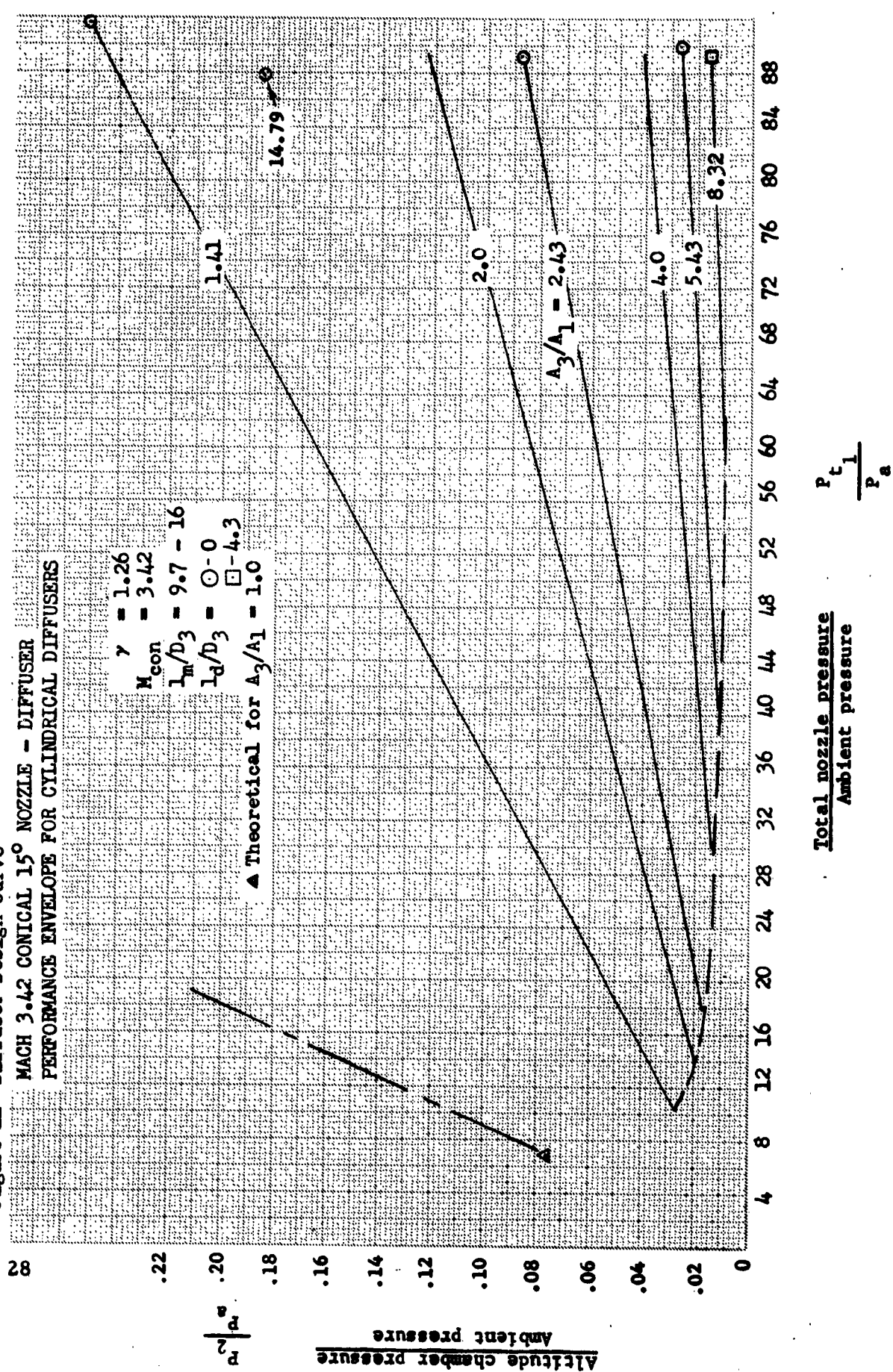
Figure 12   Diffuser Design Curve
MACH 4 CONICAL 15° NOZZLE - DIFFUSER
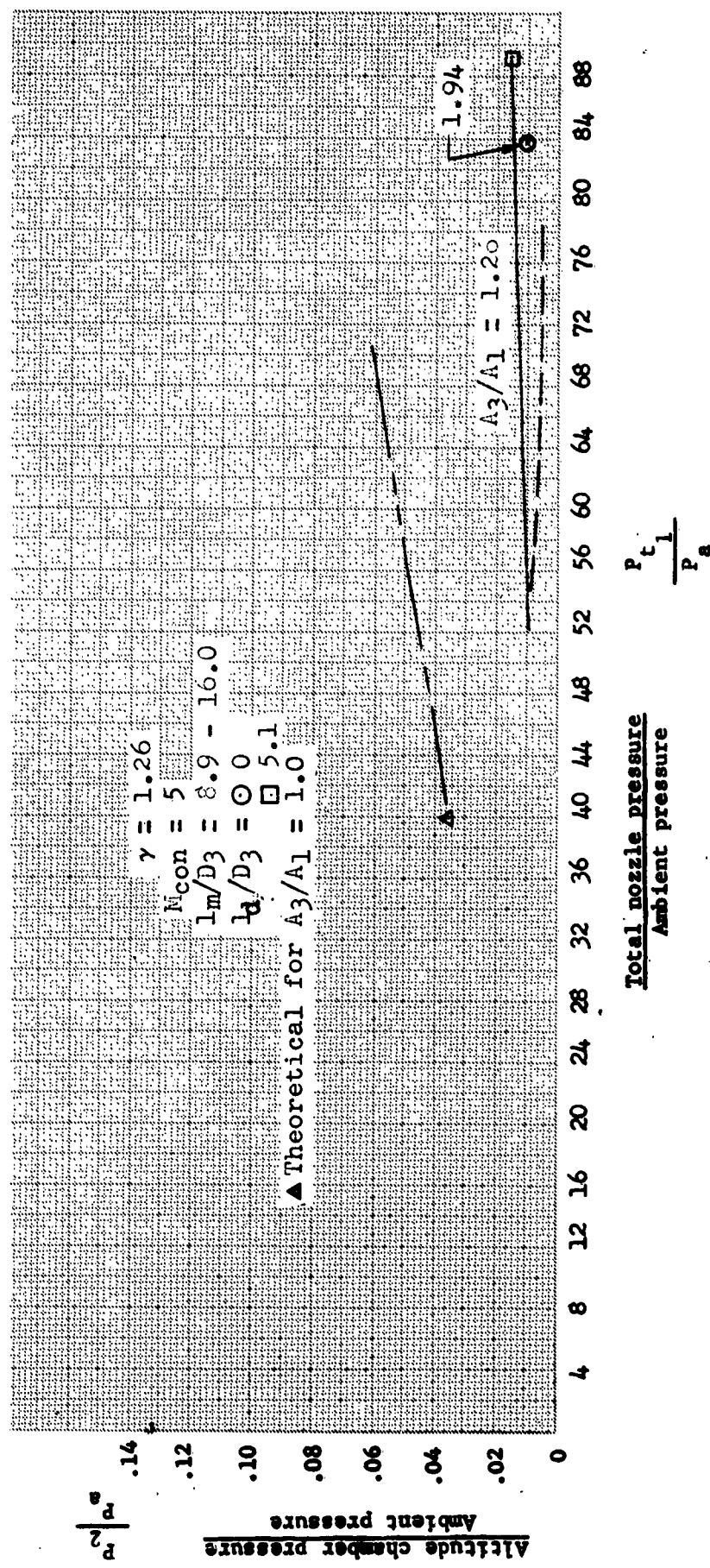PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS

$\gamma$ = 1.26
$M_{con}$ = 4
$l_m/D_3$ = 8.1 - 16
$l_d/D_3$ = ⊙ 0   □ 5.1
A$_3$/A$_1$ = 1.0

$A_3/A_1$ = 1.19

△ Theoretical for $A_3/A_1$ = 1.0

1.5
2.0

2.68
4.08
6.28

$\dfrac{P_{t_1}}{P_a}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$

$\dfrac{P_2}{P_a}$

$\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

.22  .20  .18  .16  .14  .12  .10  .08  .06  .04  .02  0

4  8  12  16  20  24  28  32  36  40  44  48  52  56  60  64  68  72  76  80  84  88

29

Figure 13 Diffuser Design Curve
MACH 5 CONICAL 15° NOZZLE - DIFFUSER
PERFORMANCE ENVELOPE FOR CYLINDRICAL DIFFUSERS

30



$\gamma = 1.26$
$M_{con} = 5$
$l_m/D_3 = 8.9 - 16.0$
$l_d/D_3 = \odot\ 0$
$\qquad\qquad \square\ 5.1$
$A_3/A_1 = 1.0$

▲ Theoretical for $A_3/A_1 = 1.0$

$A_3/A_1 = 1.20$

1.94

$\dfrac{P_{t_1}}{P_a}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$

$\dfrac{P_2}{P_a}$

$\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

Figure 14 - Diffuser Design Curve

THE SLOPE OF REGION C AS A FUNCTION OF MIXING
SECTION TO NOZZLE EXIT AREA RATIO FOR MACH 3
NOZZLES

▲ Mach 3 Isentropic Ratio of
Nozzle Exit Static Pressure
To Total Pressure

Experimental Points
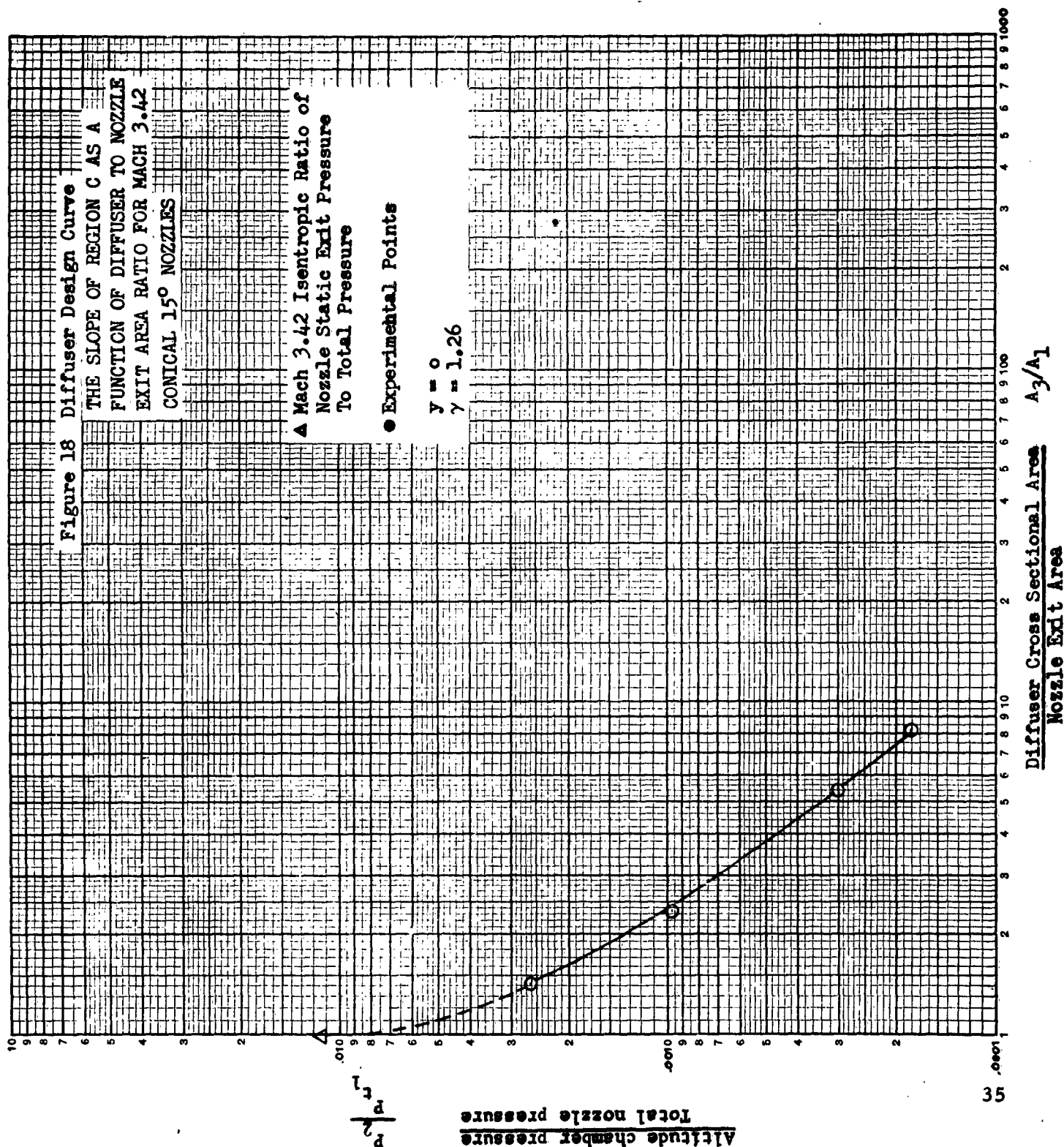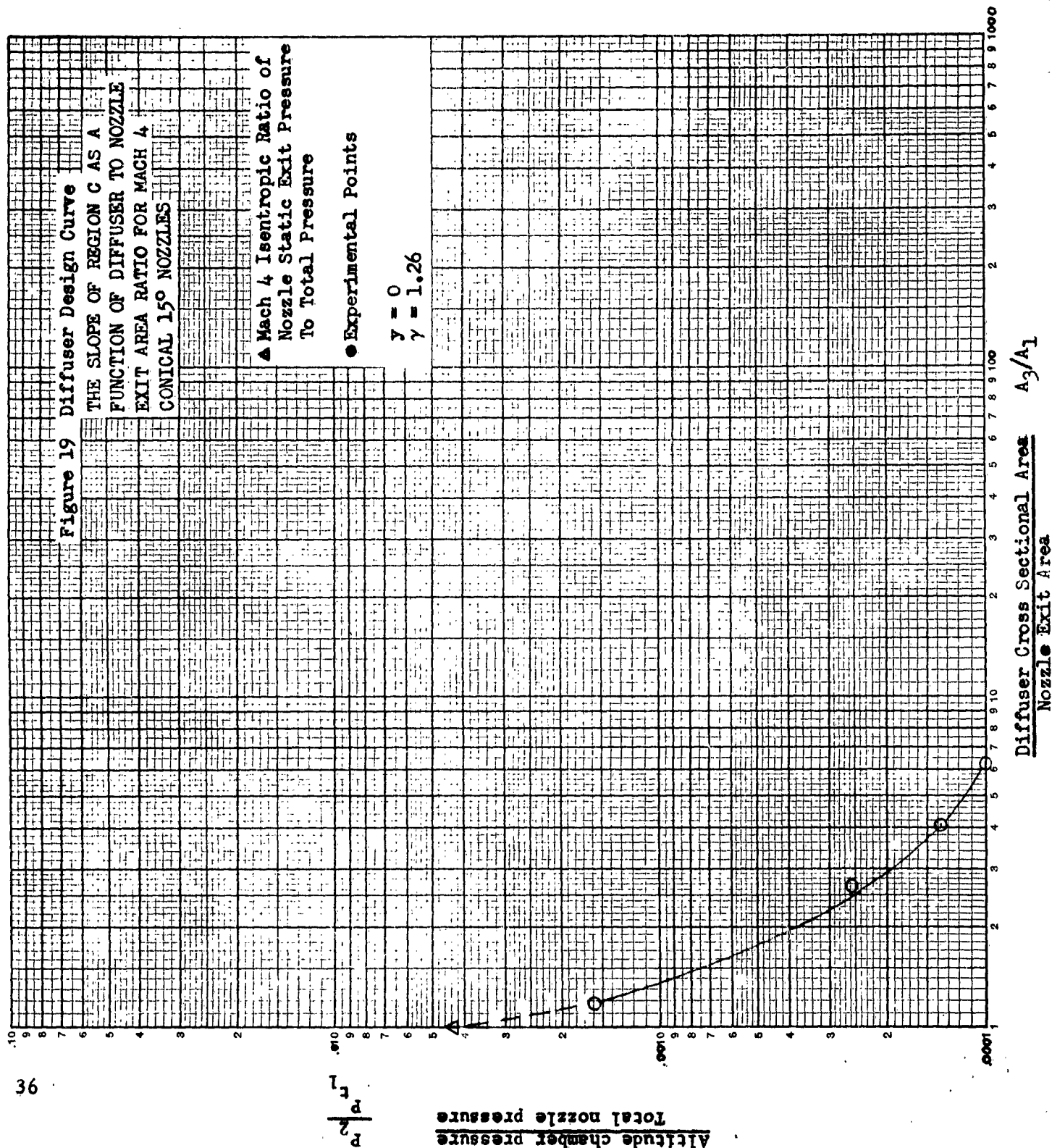⊙ Bell Shaped Nozzle
▢ 15° Conical Nozzle

$y = 0$
$\gamma = 1.4$

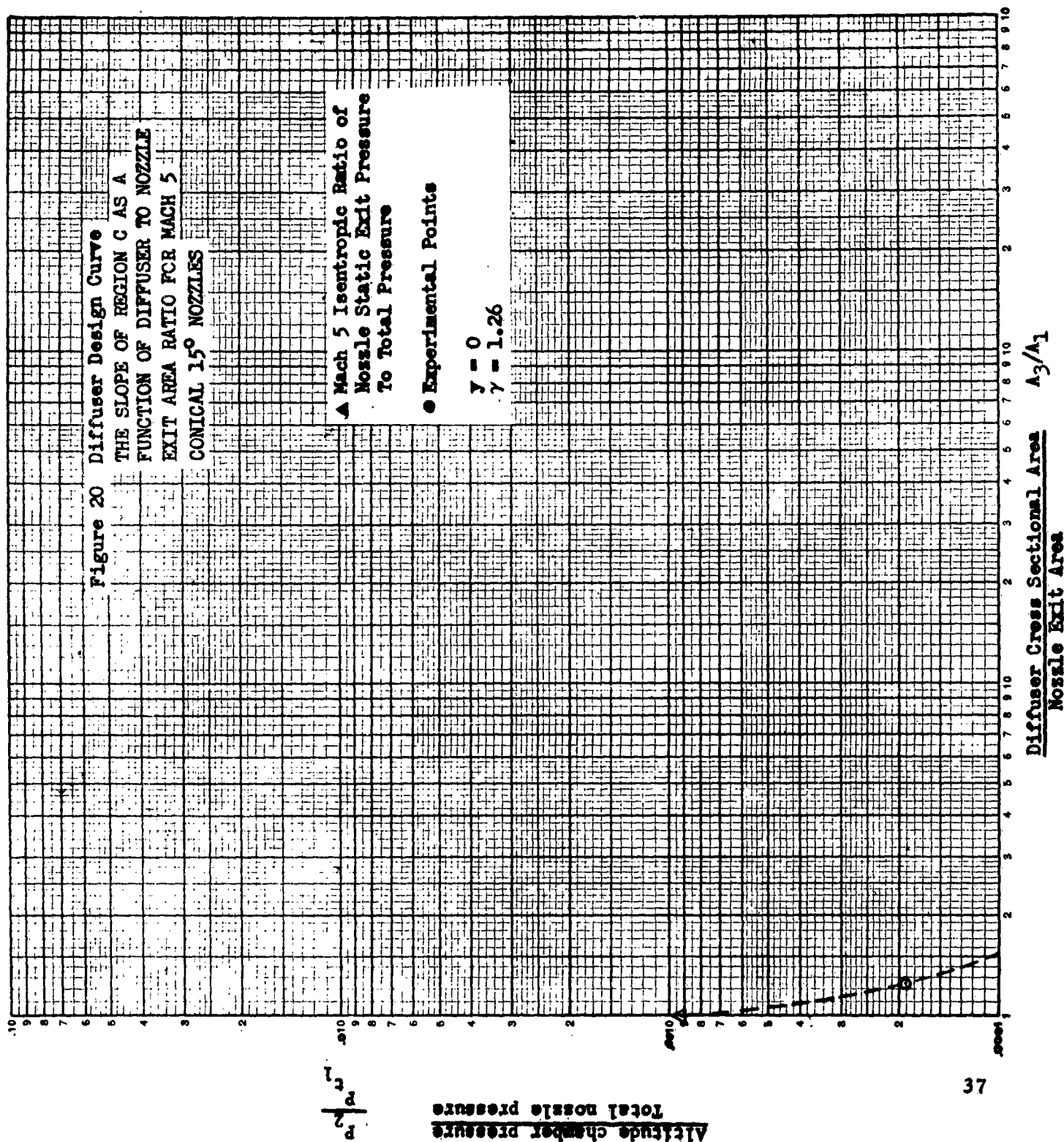$\dfrac{P_2}{P_{t_1}}$

$\dfrac{\text{Altitude chamber pressure}}{\text{Total nozzle pressure}}$

$\dfrac{\text{Diffuser Cross Sectional Area}}{\text{Nozzle Exit Area}} \quad A_3/A_1$

31
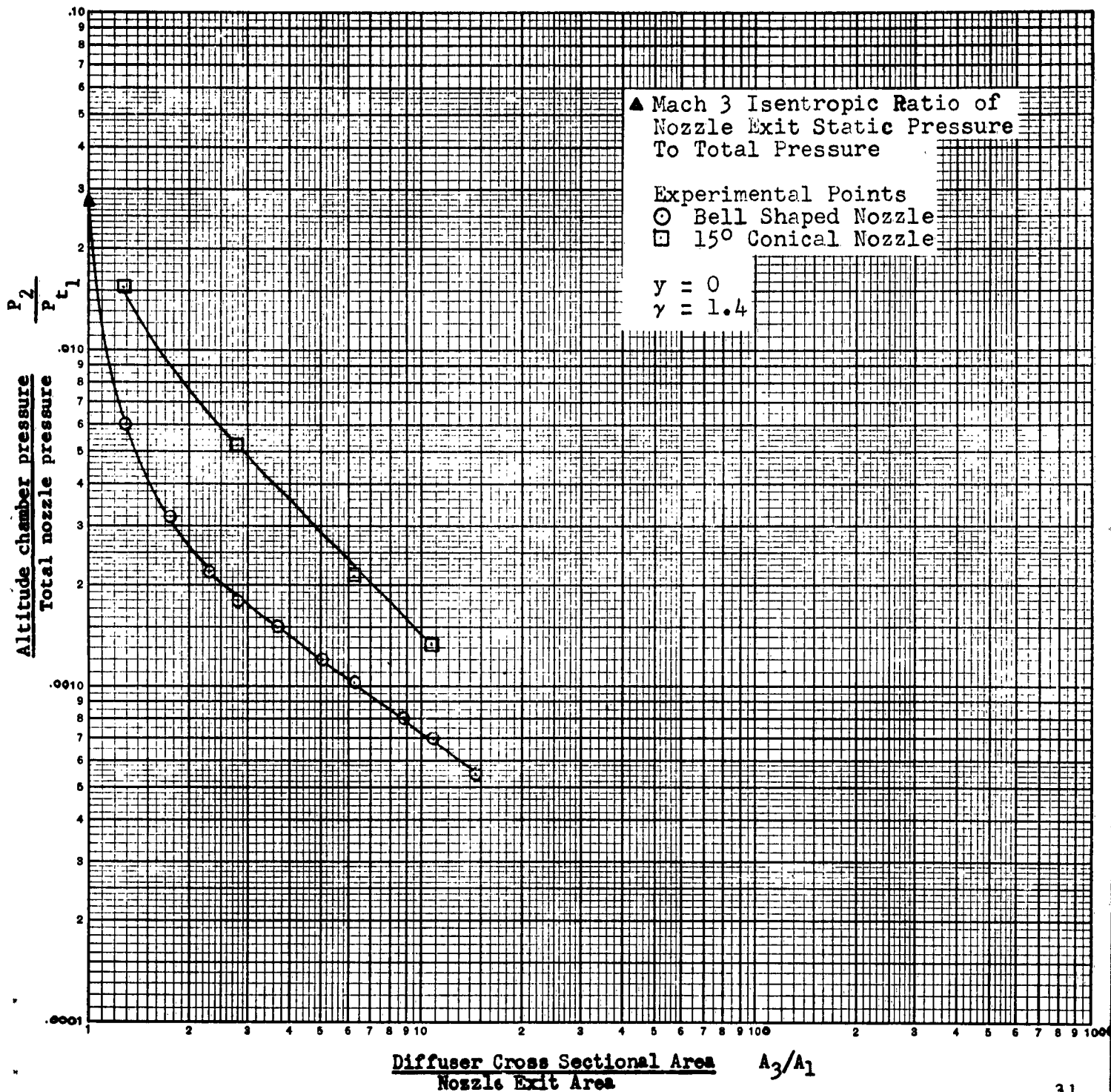
Figure 15  Diffuser Design Curve

THE SLOPE OF REGION C AS A
FUNCTION OF DIFFUSER TO NOZZLE
EXIT AREA RATIO FOR MACH 4
NOZZLES

△ Mach 4 Isentropic Ratio of
Nozzle Static Exit Pressure
To Total Pressure

Experimental Points
⊙  Bell Shaped Nozzle
☐  15° Conical Nozzle

$\gamma = 0$
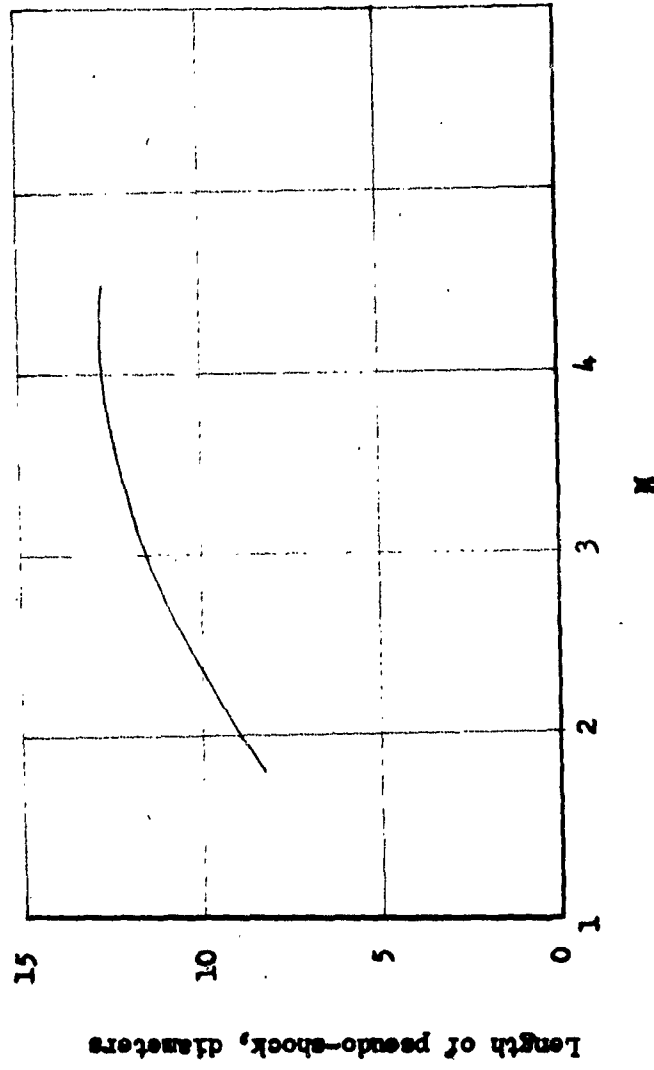$\gamma = 1.40$

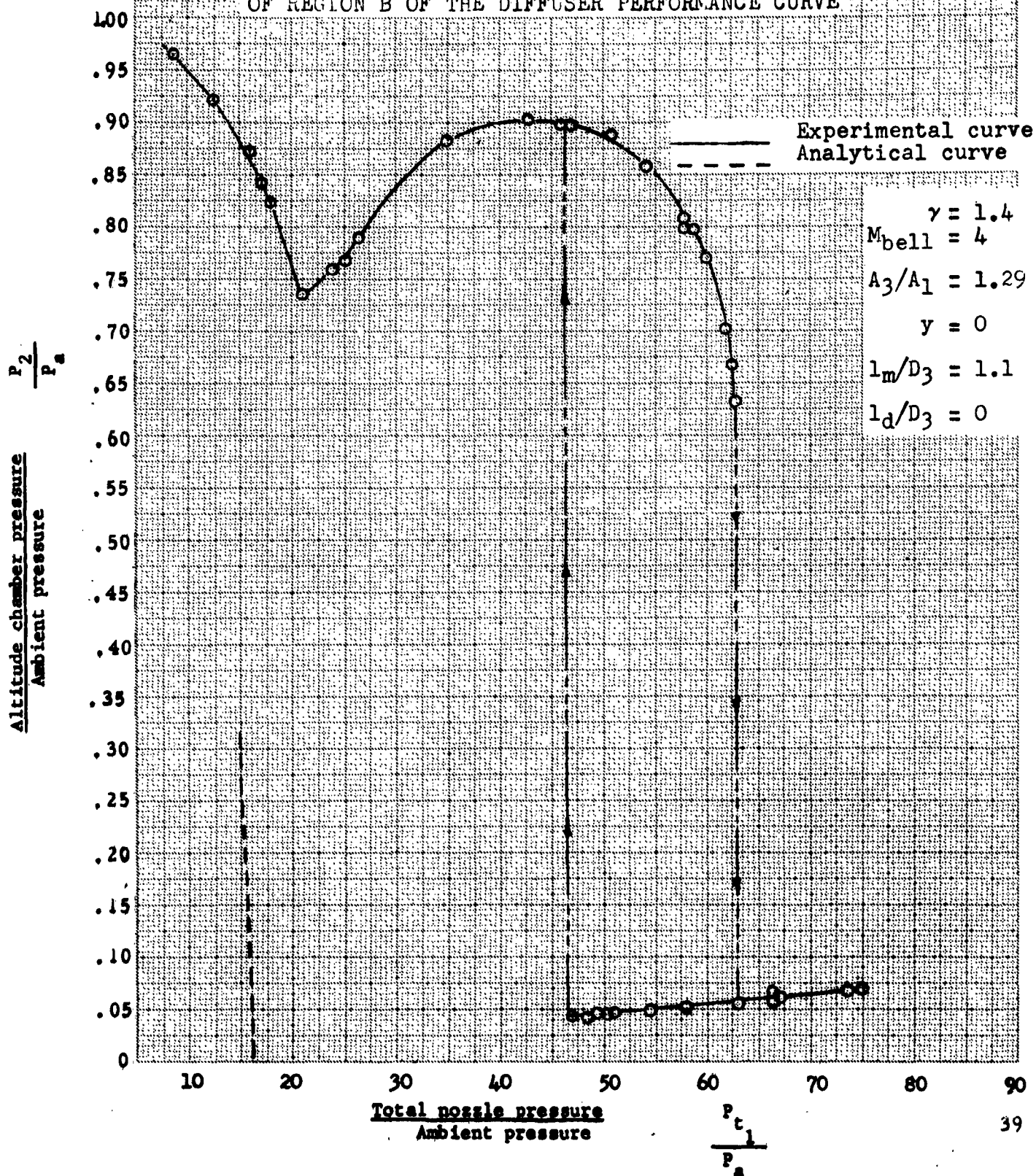Diffuser Cross Sectional Area     $A_3/A_1$
Nozzle Exit Area

$\dfrac{P_2}{P_{t_1}}$   Altitude chamber pressure
Total nozzle pressure

32

Figure 16   Diffuser Design Curve

THE SLOPE OF REGION C AS A
FUNCTION OF DIFFUSER TO NOZZLE
EXIT AREA RATIO FOR MACH 5
NOZZLES

▲ Mach 5 Isentropic Ratio of
  Nozzle Static Exit Pressure
  To Total Pressure

  Experimental Points

$y = 0$
$\gamma = 1.40$

$\dfrac{P_2}{P_{t_1}}$

Altitude chamber pressure
Total nozzle pressure

Diffuser Cross Sectional Area    $A_3/A_1$
Nozzle Exit Area

33

Figure 17 Diffuser Design Curve

THE SLOPE OF REGION C AS A
FUNCTION OF DIFFUSER TO NOZZLE
EXIT AREA RATIO FOR MACH 3
CONICAL 15° NOZZLES

▲ Mach 3 Isentropic Ratio of
Nozzle Static Exit Pressure
To Total Pressure

● Experimental Points

$y = 0$
$\gamma = 1.26$

Diffuser Cross Sectional Area $A_3/A_1$
Nozzle Exit Area

$\dfrac{p_2}{p_1}$

Altitude chamber pressure
Total nozzle pressure

Figure 18  Diffuser Design Curve

THE SLOPE OF REGION C AS A
FUNCTION OF DIFFUSER TO NOZZLE
EXIT AREA RATIO FOR MACH 3.42
CONICAL 15° NOZZLES

▲ Mach 3.42 Isentropic Ratio of
Nozzle Static Exit Pressure
To Total Pressure

● Experimental Points

$y = 0$
$\gamma = 1.26$

Diffuser Cross Sectional Area    $A_3/A_1$
Nozzle Exit Area

$\dfrac{P_3}{P_{t_1}}$

Altitude chamber pressure
Total nozzle pressure

35

Figure 19 Diffuser Design Curve

THE SLOPE OF REGION C AS A FUNCTION OF DIFFUSER TO NOZZLE EXIT AREA RATIO FOR MACH 4 CONICAL 15° NOZZLES

▲ Mach 4 Isentropic Ratio of Nozzle Static Exit Pressure To Total Pressure

● Experimental Points

$y = 0$
$\gamma = 1.26$

Diffuser Cross Sectional Area / Nozzle Exit Area   $A_3/A_1$

$\dfrac{P_2}{P_1}$   Altitude chamber pressure / Total nozzle pressure

36

Figure 20   Diffuser Design Curve

THE SLOPE OF REGION C AS A FUNCTION OF DIFFUSER TO NOZZLE EXIT AREA RATIO FOR MACH 5 CONICAL 15° NOZZLES

▲ Mach 5 Isentropic Ratio of Nozzle Static Exit Pressure To Total Pressure

● Experimental Points

$y = 0$
$\gamma = 1.26$

Diffuser Cross Sectional Area $\quad A_3/A_1$
Nozzle Exit Area

$\dfrac{p_2}{p_1}$

Altitude chamber pressure
Total nozzle pressure

37

Figure 21 (reference no. 3 )

M

Length of pseudo-shock, diameters

38

Figure 22

EXPERIMENTAL AND THEORETICAL COMPARISION
OF REGION B OF THE DIFFUSER PERFORMANCE CURVE

—————— Experimental curve
— — — Analytical curve

$\gamma = 1.4$
$M_{bell} = 4$
$A_3/A_1 = 1.29$
$y = 0$
$l_m/D_3 = 1.1$
$l_d/D_3 = 0$

$\dfrac{P_2}{P_a}$   $\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$   $\dfrac{P_{t_1}}{P_a}$

39

Figure 23

EXPERIMENTAL AND THEORETICAL COMPARISION
OF REGION B OF THE DIFFUSER PERFORMANCE CURVE

————— Experimental curve
– – – – Analytical curve

$\gamma = 1.4$
$M_{bell} = 5$
$A_3/A_1 = 1.29$
$y = 0$
$l_m/D_3 = 3.2$
$l_d/D_3 = 0$

$\dfrac{P_2}{P_a}$ — $\dfrac{\text{Altitude chamber pressure}}{\text{Ambient pressure}}$

$\dfrac{\text{Total nozzle pressure}}{\text{Ambient pressure}}$ $\dfrac{P_{t_1}}{P_a}$

40
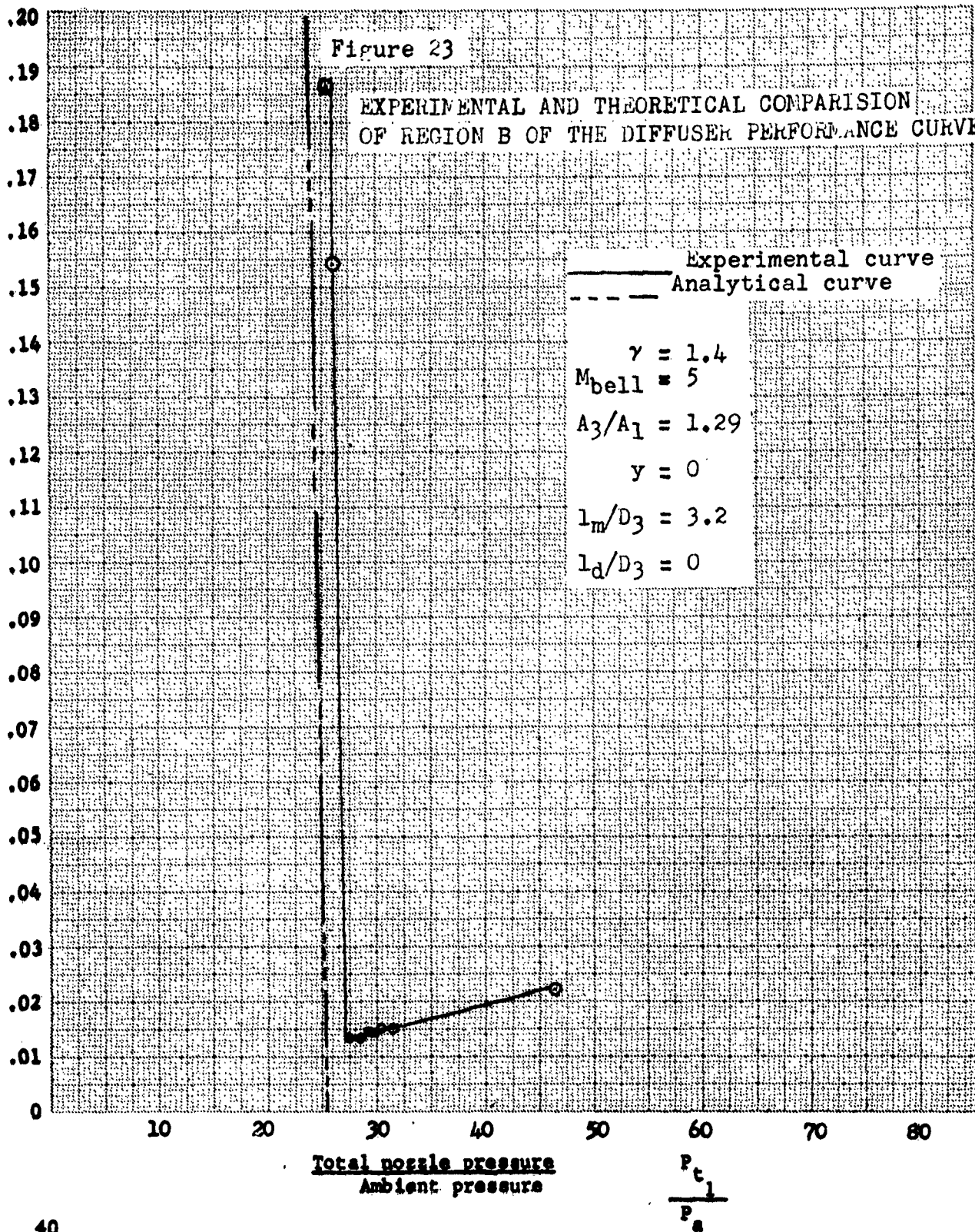
Figure 24

EXPERIMENTAL AND THEORETICAL COMPARISION OF REGION B OF THE DIFFUSER PERFORMANCE CURVE

Experimental curve
Analytical curve

$C_f = .0075$

$\gamma = 1.4$
$M_{bell} = 5$

$A_3/A_1 = 2.81$
$y = 0$
$l_m/D_3 = 11.2$

$l_d/D_3 = 0$

$\dfrac{P_2}{P_a}$  Altitude chamber pressure / Ambient pressure

Total nozzle pressure / Ambient pressure    $\dfrac{P_{t_1}}{P_a}$

41

Figure 25

Two Dimensional Variable Cross-Sectional
Area Diffuser Performance Curves
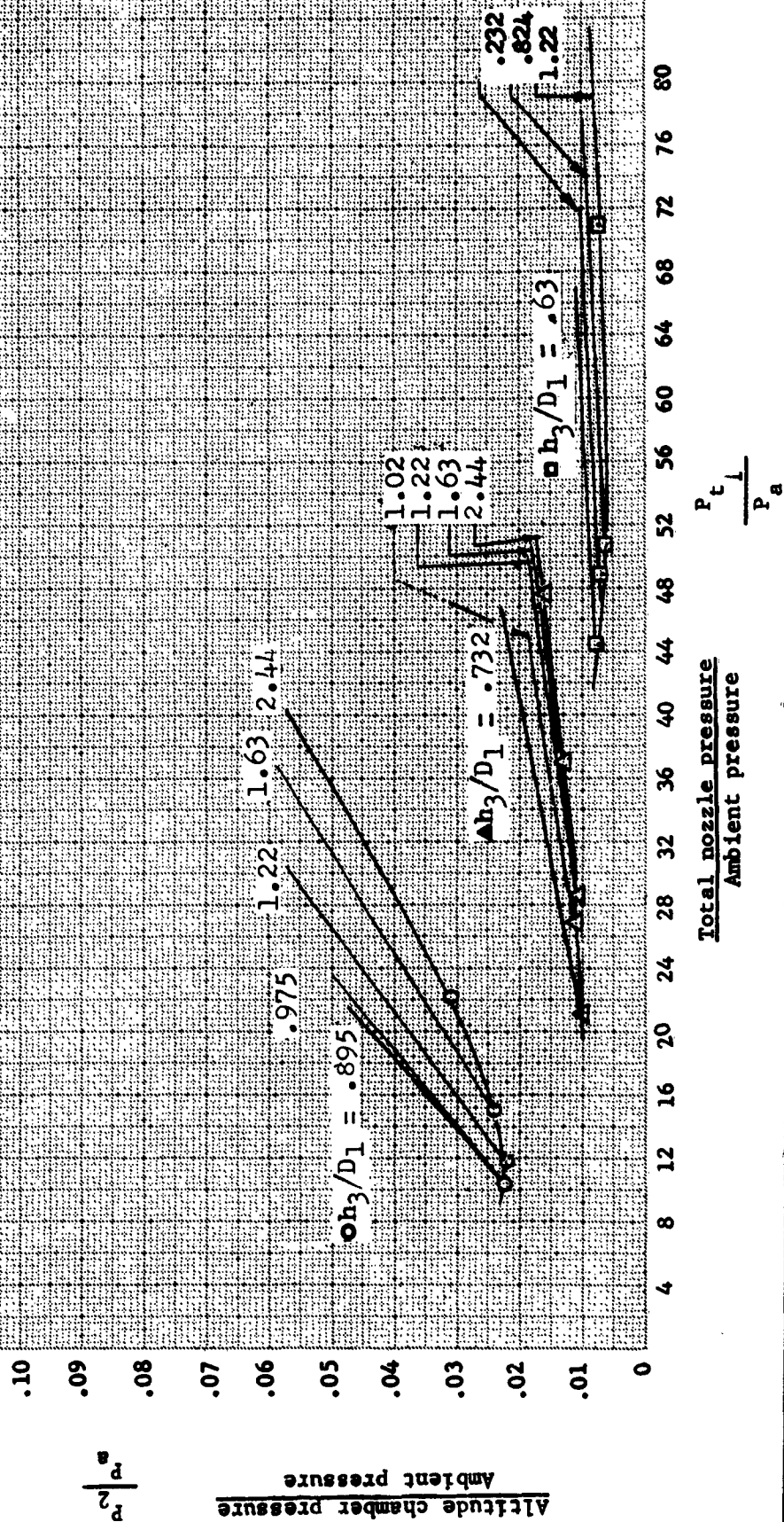
ASTIA DOCUMENT NO. AD-

Air Force Flight Test Center
Flight Test Engineering Division
Edwards AFB, California

Performance Summary of the Supersonic
Diffuser and Its Application to Altitude Test-
ing of Captive Rocket Engines. By Richard
H. Mickola, Analytical Engineer. April 1961
41 Pages. Addendum I to AFFTC-TR-60-1
   This report presents the final results of
a development program which was conducted
by the Directorate of Rocket Propulsion,
Edwards Air Force Base, California, to
determine the altitude simulation produced
by various second and third stage rocket
engines enclosed in an altitude chamber and
fired into a constant cross-sectional area

diffuser. The diffuser and supersonic nozzle configurations were tested utilizing nitrogen gas as the primary fluid; and also with 1000 pound thrust solid propellant rocket engines. The resulting data presents a method of diffuser application for captive rocket engine testing at altitudes in excess of 100,000 feet.